Modelling, Exploration and Mitigation of Partially Liquid-Filled Tanks Using Various Passive Energy Absorbers

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Abstract This study treats oscillations of a liquid in partially filled vessel under horizontal harmonic ground excitation. Such excitation may lead to hydraulic impacts applied on the tank walls. Different equivalent mechanical models are suggested to mimic the most essential sloshing regimes of the overall tank-liquid system. Then, the contribution of Nonlinear Energy Sink (NES) to the overall system mitigation is firstly examined.

We introduce the equivalent mechanical model for liquid sloshing in cylindrical tank with the well explored TMD attached **[1]**. Parameters K and C are modal stiffness and damping of the vessel fundamental (1,1) beam-type mode, respectively. The tank is of radius R and height H and exposed to arbitrary external excitation of u_g.

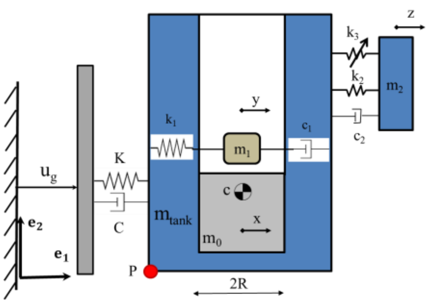

Figure 1: Scheme of cylindrical tank with liquid interacting with structure walls, and attached TMD

The liquid static and dynamic portions heights and the combined tank-static liquid portion center of gravity height are denoted by h_0 and h_1, respectively. Parameters k_2 and k_3 represent the coupling stiffness associated with the TMD and NES, respectively. c_2 is the linear damping coefficient. The PEA installation height h_2 is determined by the designer. Masses m_{tank} and m_0 are the tank shell mass and the liquid 'static' portion mass, respectively. The sloshing dynamics combines infinite number of sloshing modes with mass of m_n. However, as shown by Abramson **[2]**, the modal mass decreases rapidly with increasing mode number. Then, to reveal most important aspects of dynamics, one can take into account only the first sloshing mode and the static-like portion of the fluid in the mechanical equivalent model, as long as the excitation frequency is far from the natural frequencies of the higher modes. The normalized displacement coordinate of the sloshing mass m with respect to the tank axis is denoted by v. Impact takes place for the absolute value of v reaches unity. The liquid-structure interaction involves energy dissipation due to wave breaking and fluid viscosity, which exhibits VI behavior. Interaction between the sloshing mass and the tank walls is described by a strongly nonlinear power-form forces with high exponents potential and dissipation force functions **[3-5]**, fully-defined by empirical positive integers, which are going to be assessed both numerically and experimentally. Seismically-induced tank failure modes are explained extensively by Maekawa **[6]**. Based on the ROM, Von-Mises equivalent stresses were calculated in the tank critical point **P**. We separately apply both the well-known TMD and the cubic NES as vibration mitigation solutions. The following equations of motion are obtained for the overall tank-PEA system:

$$\begin{aligned}
&\ddot{u}+u+\varepsilon_1 v+Z\dot{u}+\varepsilon_1 Z\dot{v}-\varepsilon_2(1+\varepsilon_1)\beta_2^2 w-\varepsilon_2\kappa_2 w^3-2\varepsilon_2\beta_2\zeta_2\dot{w}=-\frac{(1+\varepsilon_1)^2}{R\Omega^2}u_{g,tt}(t)\\
&\ddot{v}+u+\left(\varepsilon_1+(1+\varepsilon_1)^2\beta_1^2\right)v+Z\dot{u}+\left(\varepsilon_1 Z+2(1+\varepsilon_1)\beta_1\zeta_1\right)\dot{v}-\varepsilon_2(1+\varepsilon_1)\beta_2^2 w-\varepsilon_2\kappa_2 w^3-2\varepsilon_2\beta_2\zeta_2\dot{w}+(1+\varepsilon_1)\kappa v^{4n+1}+(1+\varepsilon_1)\lambda\dot{v}v^{2n}=0\\
&\ddot{w}-u-\varepsilon_1\left(1+(1+\varepsilon_1)\beta_1^2\right)v+(1+\varepsilon_1)(1+\varepsilon_2)\beta_2^2 w+(1+\varepsilon_2)\kappa_2 w^3-Z\dot{u}-\varepsilon_1(Z+2\beta_1\zeta_1)\dot{v}+2(1+\varepsilon_2)\beta_2\zeta_2\dot{w}-\varepsilon_1\kappa v^{4n+1}-\varepsilon_1\lambda\dot{v}v^{2n}=0
\end{aligned} \quad (1)$$

While the TMD is examined, we take $\kappa_2 = 0$, when κ_2 is the parameter associated with the coupling between the NES and the tank structure, and in the same manner, when the NES is examined, we take $\beta_2 = 0$. The performances of both PEAs are evaluated with the help of two criteria; stress reduction and time of vibration decay. At this stage, the PEAs optimization is performed numerically; the TMD with mass about 10% of the total mass of the system allows up to 40% stress level reduction and 95% reduction of characteristic decay time in conditions of an optimal tuning.

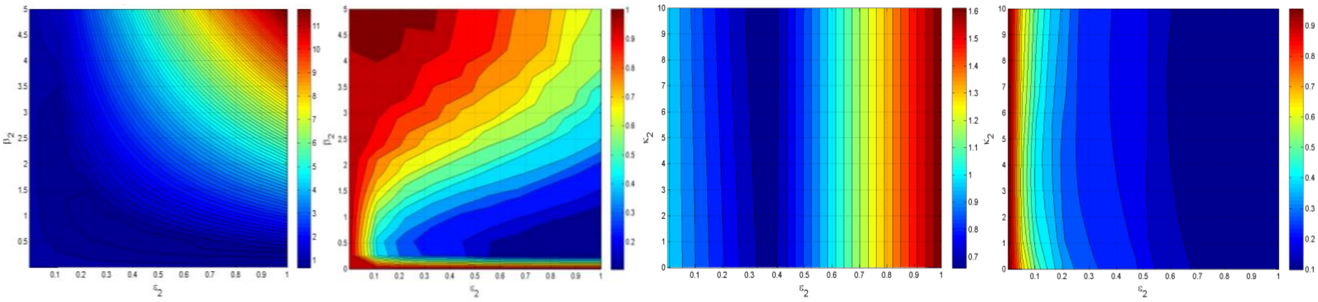

Figure 2: Example of performance optimization graph for impulsive excitation vs. PEA design parameters (stiffness and dissipation): from left to right: TMD optimization graph with respect to stress mitigation and time of decay, respectively; cubic NES optimization graph with respect to identical evaluation criteria.

Conclusions ROM is used to describe main most hazardous dynamical regimes taking place in cylindrical tank subjected to horizontal ground excitation, and internal impact regime on particular. Additional TMD and NES vibration mitigation performances were primarily examined and exhibit promising results, in term of both decay time and stresses mitigation in the tank critical location.

References

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