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From the bifurcation diagrams to the ease of playing of reed musical instruments. Application to a reed-like instrument having two quasi-harmonic resonances

J. Gilbert[1], **S. Maugeais**[2] **and C. Vergez**[3]

[1]Laboratoire d'Acoustique de l'Université du Mans - UMR 6613 CNRS
Le Mans, France
joel.gilbert@univ-lemans.fr

[2]Laboratoire Manceau de Mathématiques - Le Mans Université
Le Mans, France
sylvain.maugeais@univ-lemans.fr

[3]Aix-Marseille Univ, CNRS, Centrale Marseille, LMA, UMR 7031
Marseille, France
vergez@lma.cnrs-mrs.fr

Abstract A reed-like instrument having two quasi-harmonic resonances, represented by a 4-dimensional dynamical system, is studied using the continuation and bifurcation software AUTO. Bifurcation diagrams are explored with respect to the blowing pressure, with focus on amplitude and frequency evolutions along the different solution branches.

Wind musical instruments are nonlinear dynamical systems with a large diversity of oscillating behaviours. Among these, musicians know how to control their instrument in order to select periodic oscillating regimes, which correspond to notes when their frequency is properly adjusted. Understanding the conditions required to obtain the desired oscillating regime, in terms both of control by the musician and of the acoustic properties of the instrument is an interesting and intricate problem of nonlinear dynamics. In this study we focus on reed instruments. The use of the AUTO continuation package allows to revisit and extend the analytical approach proposed in [1].

Reed musical instruments can be described in terms of conceptually separate linear and nonlinear mechanisms: a localized nonlinear element (the valve effect due to the reed) excites a linear, passive acoustical multimode element (the musical instrument usually represented in the frequency domain by its input impedance). The linear element in turn influences the operation of the nonlinear element. The reed musical instruments are self-sustained oscillators. They generate an oscillating acoustical pressure (the note played) from a static overpressure in the player's mouth (the blowing pressure).

A reed instrument having N acoustical modes can be described as a $2N$-dimensional autonomous nonlinear dynamical system [2]. For instance a reed-like instrument having two quasi-harmonic resonances, represented by a 4-dimensional dynamical system, is considered in this study, in order to be able to use the AUTO continuation method. The modulus of the corresponding acoustic input impedance of the resonator is shown in figure 1. The acoustic input impedance is defined in the frequency domain by the ratio between the pressure and the volume flow at the input of the instrument, so that :

$$P(\omega) = Z(\omega)U(\omega), \qquad \text{where } \omega \text{ is the angular frequency.} \tag{1}$$

On the other hand, the reed-valve nonlinear behaviour can be modelled by the following polynomial nonlinearity in the time domain, where the volume flow $u(t)$ is defined as a function of the acoustic pressure $p(t)$ [2]:

$$u = u_0 + Ap + Bp^2 + Cp^3, \tag{2}$$

where u_0 is the mean volume flow and A, B and C are real numbers that depend on the control of the musician.

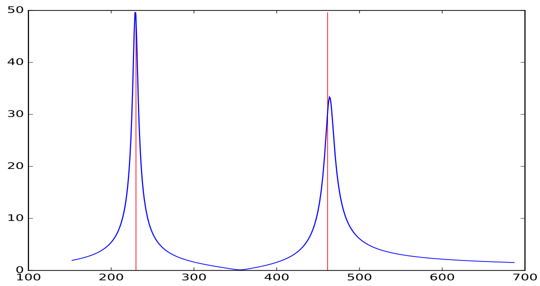

Figure 1: Modulus of the dimensionless input impedance of the resonator with respect to frequency. The two resonances are quasi-harmonic: first resonance frequency $229Hz$ (left red vertical line), second resonance frequency $463.5Hz$ slightly higher than the octave $458Hz$ (right red vertical line), corresponding to an inharmonicity of 0.012.

Bifurcation diagrams are explored with respect to the blowing pressure, with focus on amplitude and frequency evolutions along the different solution branches (see examples on figure 2). The ratio between the two acoustic resonance frequencies of the instrument (also called inharmonicity) appears to be of crucial importance.

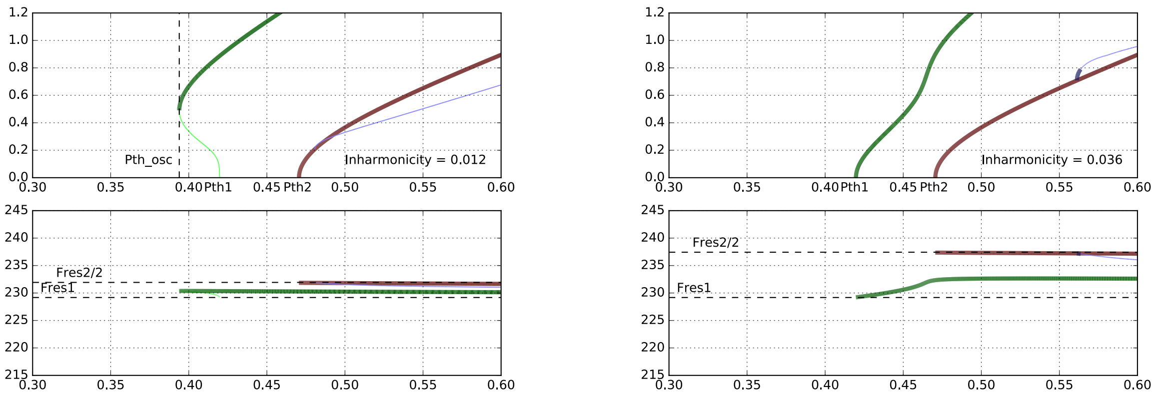

Figure 2: Bifurcation diagrams corresponding to the case of an input instrument with a 1^{st} peak 50% larger than the 2^{nd} one, like in figure 1. The ratio between the two resonance frequencies is 2.024, inharmonicity 0.012 (left) or 2.072, inharmonicity 0.036 (right). Top plots represent the amplitude of the periodic oscillation branches with respect to the blowing pressure. Bottom plots represent the frequency of the corresponding periodic solutions with respect to the blowing pressure. Green (red) lines correspond to periodic oscillations resulting from the instability of the first (second) acoustic resonance and are called 1^{st} and 2^{nd} registers respectively. Blue lines correspond to periodic oscillations of the 1^{st} register resulting from the period doubling of the 2^{nd} register.

The oral presentation will show how some of these results can be interpreted in terms of the ease of playing of the reed instrument.

References

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