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Basins of attraction of high-dimensional systems: case study of periodically excited sympodial tree

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Abstract Detecting basins of attraction is a fundamental part of a global analysis, often not performed due to the lack of analytical methods or hardware limitations of numerical techniques. In fact, what is required for numerical methods in four dimensions or more is High Performance Computing to give results in reasonable time. To overcome some of these shortcomings, we have developed the software to build basins of attraction of high-dimensional systems, illustrating its capabilities on a case study of a periodically excited three degree-of-freedom sympodial tree.

In practical applications, it is not sufficient to know only the stability of particular attractors. Supplementary information about their robustness, which is closely related to the structure of basins of attraction, is needed. This entails performing a global analysis and quantifying a basin compactness by integrity measures [1].

Our primary goal is to explore the global behavior (including practical stability) of various strongly nonlinear systems with six state-space dimensions (6D). To do so, it is necessary to employ High Performance Computing frameworks, as numerical computations of full basins in 6D is a challenging task. The Simple Cell Mapping (SCM) method [2] is an adequate choice for these computations as it offers possibilities to parallelize the most resource demanding part of basin computation - the integration of equations of motion of dynamical systems associated with a large number of initial conditions.

In this work, a software that we have developed is used as a part of a global analysis for the model of a sympodial tree with first-level branches [3] to underline the complex behaviour of basins of attraction and to show how attractors are not equally robust. This is helpful in understanding the resilience of trees to various natural conditions and environmental excitations.

The motion of the sympodial tree shown in Fig. 1 is mathematically modeled by choosing the generalised coordinates as being the absolute angles φ, ψ_1, ψ_2 (Figure 1b) and also introducing the dimensionless parameters $D_1/D = \lambda^{1/2}$, $l_1/l = \lambda^{1/2s}$, $m_1/m = \lambda^{4/3}$, $\kappa = k_1/k$, $\zeta = b/2l\sqrt{3/km}$ and $\beta = b_1/b$, leading to the following equations of motion [3]:

$$\begin{aligned}
-2\kappa(\psi_1+\psi_2) - 4\beta\zeta(\dot{\psi}_1+\dot{\psi}_2) & \\
-3\lambda^{5/3}\dot{\psi}_2^2\sin(\alpha-\varphi+\psi_2) + 2(1+\kappa)\varphi + 4(1+2\beta)\zeta\dot{\varphi} & \\
+3\lambda^{5/3}\dot{\psi}_1^2\sin(\alpha+\varphi-\psi_1) + 2(1+6\lambda^{4/3})\ddot{\varphi} & \\
+3\lambda^{5/3}\ddot{\psi}_1 cos(\alpha+\varphi-\psi_1) - 3\lambda^{5/3}\ddot{\psi}_2\cos(\alpha-\varphi+\psi_2) &= 2M\cos(\Omega t), \quad (1)\\
2\kappa\varphi + 4\beta\zeta\dot{\varphi} + 3\lambda^{5/3}\dot{\varphi}^2\sin(\alpha+\varphi-\psi_1) - 2\kappa\psi_1 & \\
-4\beta\zeta\dot{\psi}_1 - 3\lambda^{5/3}\ddot{\varphi}\cos(\alpha+\varphi-\psi_1) - 2\lambda^2\ddot{\psi}_1 &= 0, \quad (2)\\
2\kappa\varphi + 4\beta\zeta\dot{\varphi} - 3\lambda^{5/3}\dot{\varphi}^2\sin(\alpha-\varphi+\psi_2) - 2\kappa\psi_1 & \\
-4\beta\zeta\dot{\psi}_2 - 3\lambda^{5/3}\ddot{\varphi}\cos(\alpha-\varphi+\psi_2) + 2\lambda^2\ddot{\psi}_2 &= 0. \quad (3)
\end{aligned}$$

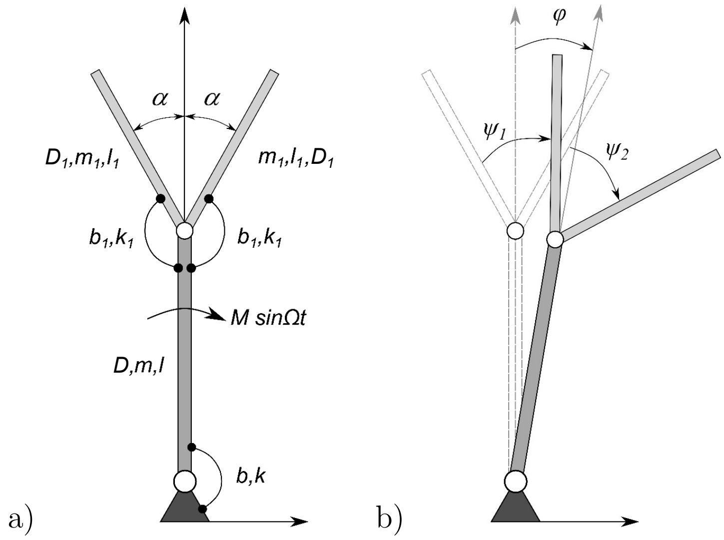

Figure 1: Model of sympodial tree with first-level branches, a) model properties, b) generalized coordinates.

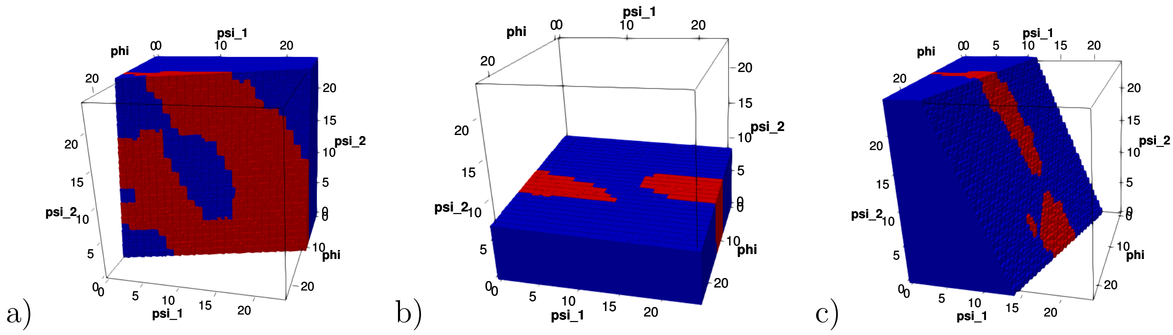

Figure 2: 3D basin cross-section $\varphi, \psi_1, \psi_1, \dot{\varphi} = 12, \dot{\psi}_1 = 11, \dot{\psi}_2 = 11$, sliced with various 2D planes.

The tree model with the parameter values $\beta = 1/2$, $\kappa = 0.3$, $\zeta = 0.03$, $\alpha = 20°$, $\lambda = 1/2$, $s = 3/2$, the excitation amplitude $M = 0.5$ and frequency $\Omega = 1.57$ has both one periodic (PA) and one quasi-periodic (QP) attractor, which coexist simultaneously. The three-dimensional cross-sections of their basin corresponding to the generalized coordinates are presented in Fig. 2 (the blue cells belong to the PA basin and the red cells to the QP one), where the initial generalized velocities are fixed at $\dot{\varphi} = 12, \dot{\psi}_1 = 11, \dot{\psi}_2 = 11$. They are sliced with various planes to underline the basins structure.

Although it is evident that the steady state of the PA is more robust than the QP one, a deeper analysis is needed to draw specific conclusions. It is required to compute integrity measures for various values of relevant system parameters, to obtain the so-called "erosion profiles" [1]. They would underline the evolution of basin regions that are considered "safe" from a practical point of view, uncovering the robustness of the examined dynamical system.

References

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