Predicting Frequency Response as Perturbation from the Conservative Limit

M. Cenedese and G. Haller

Institute for Mechanical Systems, ETH Zürich
Leonhardstrasse 21, 8092 Zürich, Switzerland
mattiac@ethz.ch georgehaller@ethz.ch

Abstract Conservative backbone curves are often observed to shape forced-damped frequency responses for multi-degree-of-freedom nonlinear vibratory systems. This relationship, however, is generally inferred a posteriori from experiments or numerical simulations. Here we discuss an analytic criterion to predict the bifurcation of frequency-amplitude plots from their conservative limits without assumptions on the amplitude or the number of degrees of freedom.

Conservative families of periodic orbits, or nonlinear normal modes, are commonly described in the analysis of nonlinear oscillations [1]. Among their properties, such oscillations are noted to act as backbones of forced-damped frequency responses. Analytic calculations supporting this observation are only available for specific, low-dimensional oscillators under the assumption of small response amplitudes.

Establishing a rigorous mathematical relation between conservative oscillations and frequency responses would have important numerical and experimental implications. Indeed, a qualitative description of the frequency response starting from conservative backbone curves would avoid computationally expensive simulations for several parameters value or shapes of the dissipative contributions. Moreover, there are experimental routines (e.g. force appropriation, [2]) that explicitly rely on the observed relation between conservative backbone curves and forced response, thus an analytic criterion could determine the range of applicability of such methods.

In this contribution, we clarify the persistence and bifurcations of conservative periodic orbits under small non-conservative perturbations [3]. We reduce the problem to the analysis of a bifurcation function that turns out to be the classic subharmonic Melnikov function [4]. When our analysis is applied to mechanical systems featuring pure forcing and arbitrary dissipative terms, it proves that either two, one or no isochronous or isoenergetic periodic orbit can arise from the conservative limit.

As an example, we apply this generalized Melnikov method to a three-degree-of-freedom system whose equations of motion read:

$$\begin{cases} \ddot{q}_1 + 3q_1 - q_2 - 0.75q_1^3 + 0.25q_1^5 = f_0\cos(\omega t) - 0.005(3\dot{q}_1 - \dot{q}_2) \\ \ddot{q}_2 + 3q_2 - q_1 - 2q_3 = -0.005(3\dot{q}_2 - \dot{q}_1 - 2\dot{q}_3) \\ \ddot{q}_3 + 4q_3 - 2q_2 + 0.5q_3^3 = -0.01(2\dot{q}_3 - \dot{q}_2) \end{cases} \qquad q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \quad . \tag{1}$$

Linearization around the origin reveals that three families of periodic orbits emanate from the origin under zero forcing and damping terms are zero. By analyzing each conservative orbit family separately, we obtain rigorous predictions for the frequency response as illustrated in the left plot of Figure 1. Using different colors for each mode, we depict the analytic relation between the maximum of the frequency response and the forcing parameter. Such relation can be extracted identifying limit points of a suitable bifurcation function evaluated using conservative trajectories only. Moreover, when these trajectories have lower amplitudes then the maximum value, two orbits laying the frequency response with the same energy bifurcate from the conservative limit, while no solution persists for higher amplitudes. We illustrate

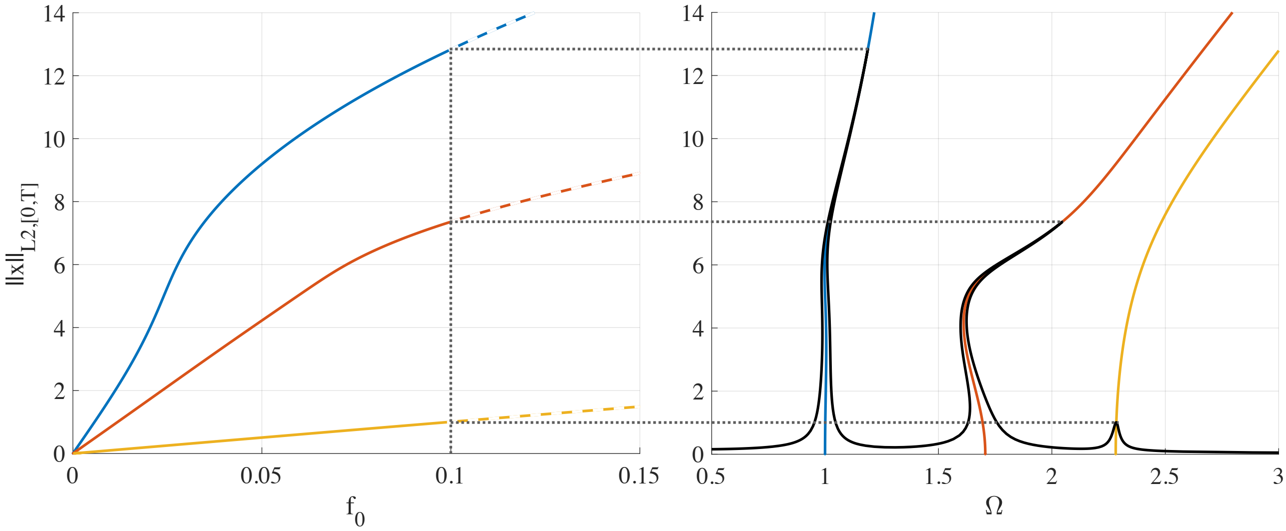

Figure 1: Left: analytic relation between the maximum value in the frequency response of the L_2-norm of the full trajectory $x = (q, \dot{q})$ and the forcing amplitude obtained from a Melnikov analysis of conservative nonlinear normal modes for the three modes; mode 1 in blue, mode 2 in orange and mode 3 in yellow. Right: the black line depicts a frequency response simulation with $f_0 = 0.1$ including three colored lines for the conservative backbone curves. The frequency is normalized with that of the first linear mode. The analytic predictions are also carried over from the left plot using gray dotted lines.

this behavior using solid and dashed lines respectively, selecting maximal modal amplitudes corresponding to $f_0 = 0.1$. Our predictions are confirmed in the right plot of Figure 1 where the black line illustrates the frequency response computed with numerical continuation. This plot is completed with conservative backbone curves with the same colors used for the left plot. Our conclusions assume that the non conservative terms are small enough and that there is no interaction between single modes.

References

[1] G. Kerschen, M. Peeters, J.C. Golinval, and A.F. Vakakis. Nonlinear normal modes, part I: A useful framework for the structural dynamicist. *Mechanical Systems and Signal Processing*, 23(1):170 – 194, 2009.

[2] M. Peeters, G. Kerschen, and J.C. Golinval. Dynamic testing of nonlinear vibrating structures using nonlinear normal modes. *Journal of Sound and Vibration*, 330(3):486 – 509, 2011.

[3] M. Cenedese and G. Haller. How do conservative backbone curves perturb into forced responses? A Melnikov's function approach. *Preprint*, 2019.

[4] V.K. Melnikov. On the stability of a center for time-periodic perturbations. *Tr. Mosk. Mat. Obs.*, 12:3 – 52, 1963.