Basic mechanisms of escape of a harmonically forced classical particle from a potential well.

Oleg Gendelman

Faculty of Mechanical Engineering, Technion – Israel Institute of Technology,
Haifa, 3200003, Israel, ovgend@technion.ac.il

Abstract Various models and systems involving the escape of periodically forced particle from the potential well demonstrate a common pattern. The minimal forcing amplitude required for the escape exhibits sharp minimum for the excitation frequency below the natural frequency of small oscillations in the well. Current work explains this regularity by detailed exploration of the transient dynamics of the escape in a number of benchmark potential wells.

Escape from the potential well under external forcing is invoked for description of numerous important processes and phenomena in physics, chemistry and engineering. Very incomplete list of such processes and phenomena includes dynamics of molecules and absorbed particles, celestial mechanics and gravitational collapse, energy harvesting, physics of Josephson junctions, transient resonance dynamics of oscillatory systems, and even such deceivingly remote topic as capsize of ships. Another important engineering phenomenon related to the escape processes is a dynamic pull-in in microelectromechanical systems.

It is known for a long time [1] that the critical force amplitude required for the escape of harmonically forced particle from various potential wells exhibits a sharp minimum at certain frequency below the frequency of small oscillations in the well. Qualitatively similar escape curves in frequency – voltage domain were observed in the problem of dynamic pull-in in MEMS excited by the alternating current.

In current work, we explain this regularity by considering the escape from three benchmark potential wells, described by the following potential functions:

$$U_0(q)=\begin{cases}-\dfrac{1}{2}+\dfrac{q^2}{2}, |q|\le 1\\ 0,\ |q|>1\end{cases};\quad U_\varepsilon(q)=\begin{cases}-\dfrac{1}{2}+\dfrac{q^2}{2}-\dfrac{\varepsilon\alpha q^4}{4}, |q|\le q_m\\ 0,\ |q|>q_m\end{cases};\ U_4(q)=\frac{q^2}{2}-\frac{q^4}{4} \qquad (1)$$

Thus, three different potential functions with increasing complexity are considered: truncated parabolic well, truncated weakly nonlinear well and strongly nonlinear quadratic-quartic potential. For the truncated parabolic well, exact solution is available and the minimum forcing required for the escape tends to zero at the resonant frequency. Addition of even small nonlinearity qualitatively modifies the dynamics. The minimum escape forcing becomes nonzero and shifts towards smaller frequencies. Interestingly, strongly nonlinear model exhibits quite similar properties, at least in the vicinity of main resonance. For the nonlinear wells, the analytic treatment relies on the approximation of isolated resonance [2] and is performed with the help of appropriate transformations to action-angle variables. Results for the escape thresholds for the nonlinear wells are presented in Figure 1.

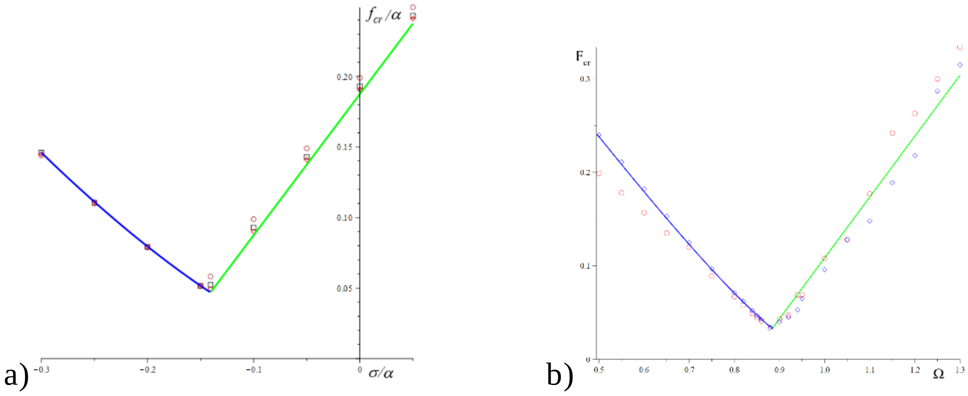

Figure 1 Escape threshold versus the excitation frequency; a) weakly nonlinear truncated well; b) strongly nonlinear well. Dots, circles and diamonds stay for numeric results.

For both nonlinear models one reveals two qualitatively different scenarios of the escape transition. They are illustrated for the strongly nonlinear well in Figure 2.

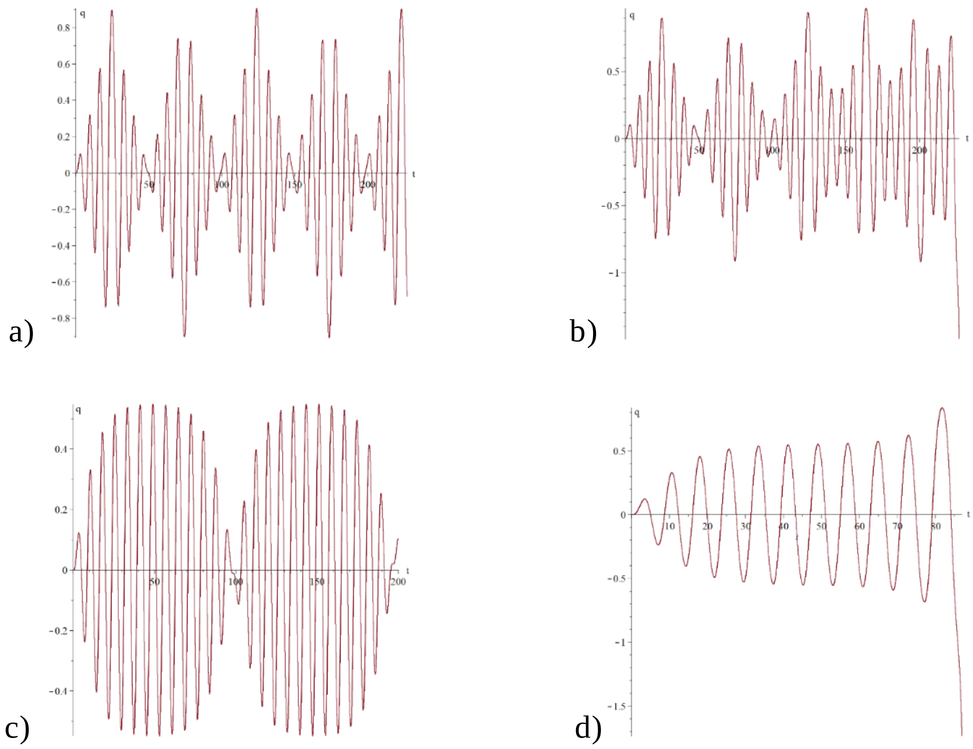

Figure 2. Escape mechanisms: a)-b) –the maximum, c) –d) – the dynamical saddle.

Both in transitions 2a)-2b) and 2c)- 2d) the forcing amplitude differs by about 1%. Still, in the latter case the maximum response energy immediately before the escape achieves only about a half of the well depth. The abrupt increase occurs due to a dynamical heteroclinic bifurcation that reveals itself when details of the transient escape dynamics are considered.

References

[1] L.N.Virgin, R.H. Plaut, C.C. Cheng, *Prediction of Escape from a Potential Well under Harmonic Excitation.* International Journal of Non-Linear Mechanics. 21, 357-365, 1992**.**

[2] O.V.Gendelman, *Escape of a harmonically forced particle from an infinite-range potential well: a transient resonance.* Nonlinear Dynamics. 93, 79-88 2018.