Targeted nonlinear energy transfer for electroacoustic absorbers

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Abstract An electroacoustic loudspeaker linearly coupled to an electric nonlinear shunt circuit acting as a nonlinear energy sink is considered. An analytical treatment enabling to analyze the behavior of the system around the 1:1 resonance at different time scales is performed. Extended form of Manevitch's complex variables is introduced, taking into account higher harmonics. Periodic and strongly modulated responses are well predicted.

We consider an electroacoustic loudspeaker, shunted to an electrical nonlinear circuit (cubic nonlinearity is chosen to use the nonlinear energy sink concept developed in [1]) and subjected to an external periodically varying sound pressure. The dynamics can be described by the following equations [2]:

$$\begin{cases} M_{ms}\ddot{\boldsymbol{x}}(t) + R_{ms}\dot{\boldsymbol{x}}(t) + C_{mc}^{-1}\boldsymbol{x}(t) - CBl\dot{\boldsymbol{V}}_{\boldsymbol{c}}(t) = SA_m\cos(\omega t), \\ C(L_e + L_c)\ddot{\boldsymbol{V}}_{\boldsymbol{c}}(t) + C(R_e + R_c)\dot{\boldsymbol{V}}_{\boldsymbol{c}}(t) + k\boldsymbol{V}_{\boldsymbol{c}}^3 + Bl\dot{\boldsymbol{x}}(t) = 0, \end{cases} \tag{1}$$

where $\boldsymbol{x}$ and $\boldsymbol{V_c}$ describe the small displacement of the loudspeaker membrane and the electrical potential applied to the capacitor in the nonlinear shunt circuit, with $\dot{\boldsymbol{x}}(t) = \frac{d\,x(t)}{dt}$. M_{ms}, R_{ms} and C_{mc} are the mass, the mechanical resistance of the moving bodies and the equivalent compliance of the enclosed loudspeaker. Bl is the force factor of the transducer, B represents the magnetic field magnitude and l stands for the length of the wire in the voice coil. A_m stands for the pressure amplitude, ω the angular frequency and S the diaphragm area. From the electrical side, R_e and L_e are respectively the DC resistance and the inductance of the voice coil and $Bl\dot{\boldsymbol{V}}_{\boldsymbol{c}}(t)$ is the back electromotive force. R_c, L_c and C are the inductor, resistor and capacitance of the corresponding nonlinear shunt circuit. k is the nonlinear coefficient (related to the design of the electronic circuit). Then we introduce the following non-dimensional time variable $T = \omega_0 t$ with $\omega_0 = \sqrt{1/(M_{ms} + C_{mc})}$ and $\Omega = \omega/\omega_0$. We denote $\boldsymbol{x}'(t) = \frac{d\,x(t)}{dT}$. Then, scaling of parameters is also done by considering their physical range and by expressing them in function of a small parameter $\varepsilon = L_e + L_c \ll 1$.

The Slow Invariant Manifold (SIM) of the system is generally obtained by treating the system (1) analytically after introducing the classical Manevitch's complex variables [3]. However, with the present system, we can show (by looking at results by direct numerical integration of the system) that the contribution of the third harmonic for $d\boldsymbol{V_c}/dT$ in the transient regime is not negligible and that the contribution of the third harmonic for $d\boldsymbol{x}/dT$ in the transient regime is negligible. That is why we propose to extend the method by introducing higher harmonics of the present system:

$$\begin{cases} \boldsymbol{x}' + i\Omega \boldsymbol{x} = \varphi_{11}(T)e^{i\Omega T}, \\ \boldsymbol{V_c}' + i\Omega \boldsymbol{V_c} = \varphi_{21}(T)e^{i\Omega T} + \varphi_{23}(T)e^{3i\Omega T}. \end{cases} \tag{2}$$

We also write the complex variables into their polar form as $\varphi_{jm} = N_{jm}e^{i\delta_{jm}}$.

The analytical treatment allows to detect time multi-scale energy pumping between the primary system that describes the displacement of the loudspeaker and the shunt nonlinear circuit. It permits the detection of the SIM of the system at fast time scale, in addition to the equilibrium and fold singularities identification of the obtained reduced order system at slow time scales. Figure 1 (a) illustrates the fact that the extended method allows to better predict the SIM than the classical one.

Figure 1 (b) shows the normalized admittance according to frequency for different cases of coupling. The classical shunt optimal resistor permits a significant decrease in the normalized admittance with a perfect absorption at the resonance. However, this approach is limited to a narrow range of frequency with no possible broadening control of the bandwidth. In the vicinity of 1:1 resonance, an optimal response frequency of the system can be identified through a selected threshold. It corresponds to the maximum of energy that the primary system can reach during an energy exchange process with the NES. Thus, the optimal design defined in terms of normalized admittance is represented by the horizontal line. The added passive nonlinear shunt circuit allowed a significant decrease of the admittance, principally at the vicinity of the resonance frequency where the targeted energy transfer prevents the velocity to exceed a certain amplitude. Moreover, we can identify that the frequency bandwidth undergoes a 45% of relative increase.

(a) (b)

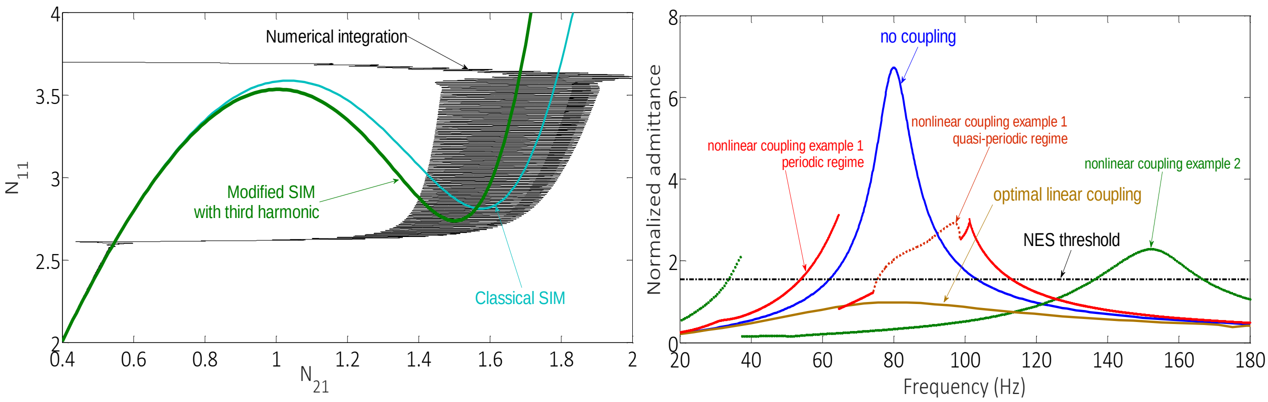

Figure 1: (a) Comparison between the analytical classical SIM, the new one obtained by taking into account the third harmonic and the direct numerical integration of the initial system; (b) Comparison of the normalized admittance as function of frequency between the cases of open circuit, optimal linear resonator and the shunt nonlinear circuits with two examples of nonlinear coupling (two different amplitudes for sound incident wave).

References

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