

Dynamics of a Rotating Nonlinear Hub-Beam Structure

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Abstract Dynamics of a beam attached to a rotating rigid hub is presented in the paper. A beam model, based on extended Bernoulli-Euler theory, takes into account a nonlinear curvature, coupled transversal and longitudinal oscillations and non-constant angular velocity of the hub. Natural and forced vibrations are studied on the basis of exact equations of motion and associated dynamic boundary conditions derived from Hamilton principle.

The development of modern materials and design of lightweight and flexible rotating structures give rise to better understanding their dynamic response. Thus, more precise mathematical models which take into account nonlinearity of the system are required. Derivation of equations of motion of nonlinear flexural-torsional vibrations of a beam has been presented in [1] where a nonlinear beam curvature has been taken into account. However, inextensibility condition has been applied in the mathematical formulation. This resulted in a constraint relation between transversal and longitudinal displacements. Similar approach to the rotating structure with attached tip mass is presented in [2], but additionally quasi-static elongation of the beam and tension force occurring due to beam rotation have been considered. A rotating thin-walled composite beam has been studied in [3] but because of structural complexity only a linear deformation field has been included in the mathematical description. Nonlinear vibrations of a rotating cantilever beam have been studied recently in [4] where hardening or softening phenomenon has been demonstrated for reduced nonlinear Bernoulli-Euler beam based on the inextensibility condition.

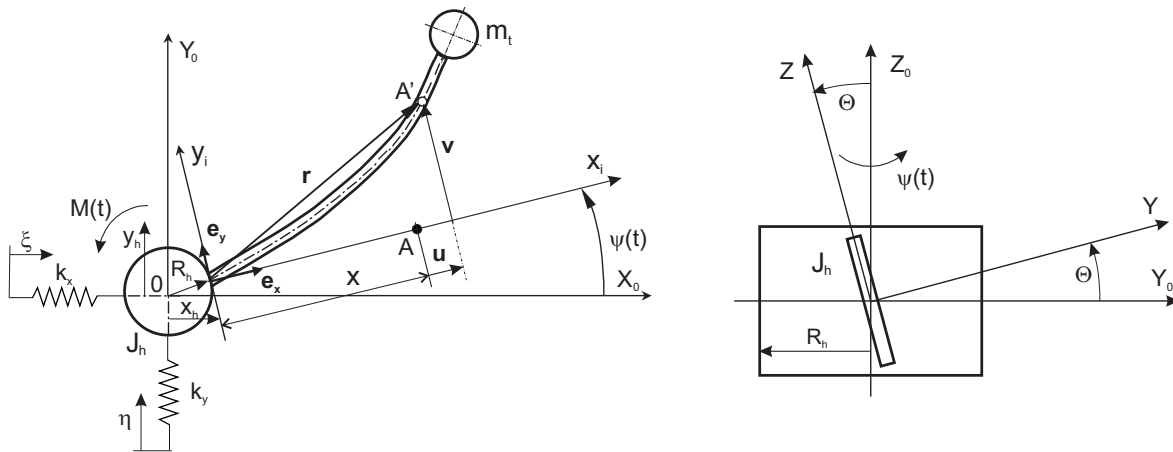


Figure 1: A model of a rotating hub-beam system with attached tip mass m_t , (a) top view with indicated deformations and (b) side view with preset angle Θ .

In the present paper we will develop a model of a rotating beam presented in [5] however, apart from rotation of the hub also translation of its center is considered. Equations of motion are based on assumptions of Bernoulli-Euler beam theory extended for axial elongation of the beam and nonlinear curvature based on the strict definition.

The model with global and local coordinate sets as well as displacements of an elementary beam point (u, v) are presented in Fig. 1(a) while the beam preset angle Θ is shown in side

view (Fig. 1b). The strain of the elementary segment and beam's curvature are defined as:

$$\varepsilon = \sqrt{(1 + u')^2 + v'^2} - 1, \quad \kappa = \frac{\partial \phi}{\partial s} = \frac{v''(1 + u') - v'u''}{[(1 + u')^2 + v'^2]^{3/2}} \quad (1)$$

Components of velocity vector $\dot{\mathbf{R}}$ of an arbitrary beam point in the absolute coordinates frame is defined as:

$$\begin{aligned} \dot{R}_{X_0} &= \dot{X}_h - \left[(R_h + s + u) \dot{\psi} + \dot{v} \cos \Theta \right] \sin \psi - v \cos \Theta \dot{\psi} \cos \psi \\ \dot{R}_{Y_0} &= \dot{Y}_h - \left[(R_h + s + u) \dot{\psi} + \dot{v} \cos \Theta \right] \cos \psi - v \cos \Theta \dot{\psi} \sin \psi \\ \dot{R}_{Z_0} &= \dot{v} \sin \Theta \end{aligned} \quad (2)$$

The differential equations of motion are derived on the basis of the extended Hamilton principle of least action

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W_{nc}) dt = 0 \quad (3)$$

where T and V are kinetic and potential energies

$$T = \frac{1}{2} J_h \dot{\psi}^2 + \frac{1}{2} \int_0^L \rho_1 \dot{R}^2 ds + \frac{1}{2} m_t \dot{R}_t^2, \quad V = \frac{1}{2} \int_0^L (EI \kappa^2 + EA \varepsilon^2) ds + \frac{1}{2} k_X (X_0 - \xi)^2 + \frac{1}{2} k_Y (Y_0 - \eta)^2 \quad (4)$$

and δW_{nc} is virtual work of other nonconservative forces.

Substituting definitions for the strain, curvature and velocity into potential and kinetic energies as well as considering nonconservative forces, damping and excitations, we derive partial differential equations (PDEs) of motion of the rotating nonlinear hub-beam structure and associated dynamical boundary conditions for transversal and longitudinal vibrations. Due to long and complex forms the equations are not presented in the abstract. The ongoing work is to solve the equations by the direct attempt to PDEs by the multiple time scale method and then to determine natural and forced vibrations for fixed and varied angular velocity.

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