Edge states and frequency response in nonlinear model of forced-damped valve spring

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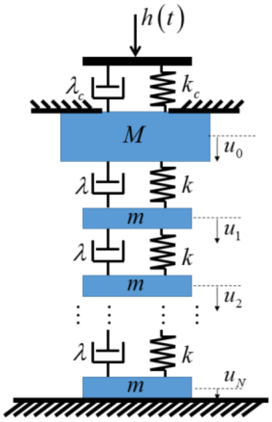
Abstract The nonlinear dynamics of the valve spring of an internal combustion engine is mathematically modeled and investigated. Exact solutions in the form of time-periodic and spatially localized edge states are derived. The stability of the system is analyzed using the Floquet theory. Comparison of the analytical solution with numerical simulations and experimental test results conducted on an actual valve spring yields an excellent agreement.

Nonlinearity and discreteness are inherent in many systems in nature, e.g. Josephson junction networks, Bose-Einstein condensates, micro-mechanical devices and optical devices. This interplay between nonlinearity and discreteness supports time-periodic and spatially localized solutions which are often referred to as discrete breathers (DBs) [1].

Exact solutions for symmetric discrete breathers are derived for the Hamiltonian model [2] and for the case of a homogenous external forcing with restitution coefficient less than unity, where it is the only source of damping in the model [3]. The stability of the periodic solutions is investigated using Floquet's theory by observing the movement of the eigenvalues of the monodromy matrix in the complex plane [4].

The purpose of our current work is to develop an exact solution and stability threshold for edge states in finite non-homogenous forced-damped chain with vibro-impact nonlinearity.

The proposed model given in Figure 1 presents a finite non-homogenous one-dimensional mass-spring-damper discrete chain. Periodic displacement (shown in Figure 2), which mimics the actual camshaft profile, is applied to the upper end of the chain while the other end is fixed. The zeroth mass experiences an impact that satisfies the Newton impact law with restitution coefficient less than unity. There are two damping sources in this model, one at the contact and the other due to internal damping of the spring material. The nonlinearity of the model originates from the periodic impact interactions.



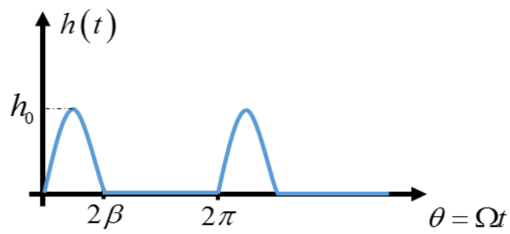


Figure 2: The applied displacement mimics a general Camshaft profile.

Figure 1: The nonlinear mass-spring-damper chain under periodic excitation.

Considering the periodicity of the excitation $h(t)$, its hybrid behavior can be modeled as a continuous form using Fourier Series as follows:

$$h(t) = a_0 + \sum_{r=1}^{\infty} \left[a_r \cos(r\Omega t) + b_r \sin(r\Omega t) \right] \tag{1}$$

Following Gendelman [3], the non-smooth bounding condition at the impact can be eliminated by representing it as an external loading force:

$$\begin{aligned} &\mathrm{M}\ddot{u}_0 + \lambda(\dot{u}_0 - \dot{u}_1) + k(u_0 - u_1) + k_c u_0 + \lambda_c \dot{u}_0 = \lambda_c \sum_{r=1}^{\infty} r\Omega\left[-a_r \sin(r\Omega t) + b_r \cos(r\Omega t)\right] + \\ &+ k_c\left[a_0 + \sum_{r=1}^{\infty}\left[a_r \cos(r\Omega t) + b_r \sin(r\Omega t)\right]\right] + 2p\sum_{j=-\infty}^{\infty} \delta\left(t - \phi + j\frac{2\pi}{\Omega}\right) \\ &m\ddot{u}_n + \lambda(2\dot{u}_n - \dot{u}_{n-1} - \dot{u}_{n+1}) + k(2u_n - u_{n-1} - u_{n+1}) = 0 \\ &u_N = 0 \end{aligned} \tag{2}$$

where $2p$ represents the change in the linear momentum, ϕ is the phase lag between the external forcing and the impact and $\delta(x)$ is the Dirac-delta function.
The edge state solution of Equation (2) has the following form:

$$\begin{aligned} &u_n = u_{n,0} + \sum_{r=1}^{\infty} C_{n,r} \cos\left(r\Omega(t-\phi)\right) + \sum_{r=1}^{\infty} S_{n,r} \sin\left(r\Omega(t-\phi)\right) \\ &u_{n,0} = U_0\left(1 - \frac{n}{N}\right); \\ &C_{n,r} = A_r f_r^n + B_r f_r^{-n}; \quad S_{n,r} = D_r g_r^n + E_r g_r^{-n} \end{aligned} \tag{3}$$

The obtained solution is characterized by a strongly localized at the edge of the chain as shown in Figure 3. Finally, the analytical solution is compared with numerical simulations and experimental data obtained from analysis of actual automotive valve spring (Figure 4).

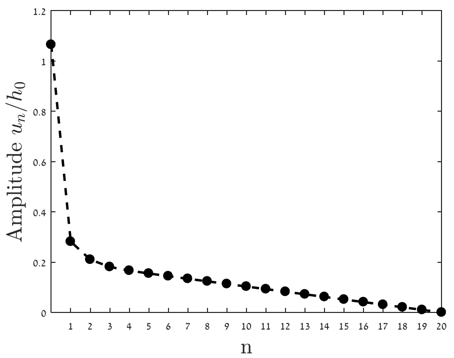


Figure 3: The edge state profile.

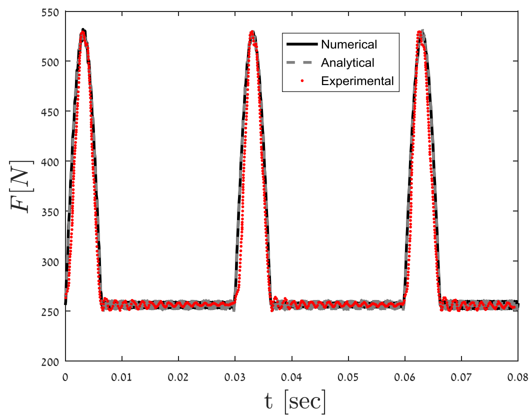


Figure 4: Spring force: comparison of the exact solution with numerical simulations and experimental test results.

References

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