

# Non linear vibration of strings against obstacles in plucked string instruments

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**Abstract** Contacts occurring during the string vibration provide a non linear effect which shapes a specific tone and contributes to the sound identity of the instrument. A conservative scheme based on a modal representation of the string displacement is presented, allowing the simulation of a stiff, damped string vibrating against an obstacle with an arbitrary geometry. Applications on the tanpura and the medieval harp with one contact point and on the electric bass with numerous contact points are proposed.

Collisions in musical string instruments play a fundamental role in the sound production in numerous instruments. These collisions are either expected, for specific musical instruments design or instrumentalist gesture, or undesired and related to adjustment issues. Taking into account these collisions is then essential to insure the realism of the sound simulation of musical instruments. An abundant literature [1] exists on numerical simulations of a string vibrating against an obstacle but a few studies include the two transverse polarisations, physical parameters of the string and a comparison with experimental data. The talk aims at presenting a numerical tool to simulate musical strings vibrating against a unilateral distributed obstacle, and confronting it to experiments in detail for various instruments.

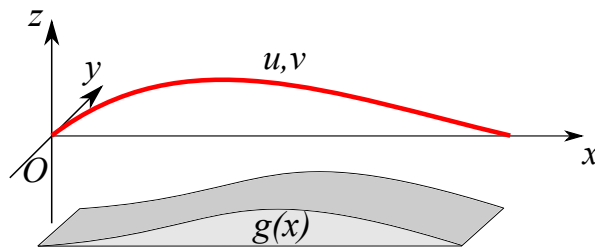


Figure 1: Drawing of a string vibrating against a unilateral obstacle described by  $g(x)$ .

The numerical method is based on a modal description of the string considered stiff, damped and simply supported at both ends [2]. The string collides with an obstacle having a profile  $g(x)$  which is constant along  $(Oy)$ . The contact force is modeled by a penalty approach where a small amount of interpenetration is allowed. Note also that an alternative modeling using nonsmooth numerical integration has also been investigated [3], showing equivalent results for the considered cases. Along the  $(Oy)$ -polarisation, a friction force is selected as a regularized empirical Tresca friction law [4]. In order to take into account the vibrations of the instrument body, the mobility at the string ends is then added to complete the model. In the case of solid-body electric guitars and basses, it has been shown that the bridge mobility is negligible as compared to that at the nut [5].

For the experimental approach, the mobility at the nut and the string characteristics (modal parameters, damping, stiffness) have to be obtained to feed the numerical model. String characteristics are identified using the ESPRIT algorithm from the measured vibration of the string stretched on a frame guaranteeing rigid-end conditions [5]. In order to compare numerical results to experimental ones, both displacements along ( $Oz$ ) and ( $Oy$ ) are recorded simultaneously, with optical sensors calibrated according to the procedure described in [6]. The profile  $g(x)$  is also measured using a ruler or an optical profilometer. The initial condition is provided by pulling the string with a copper wire until it breaks.

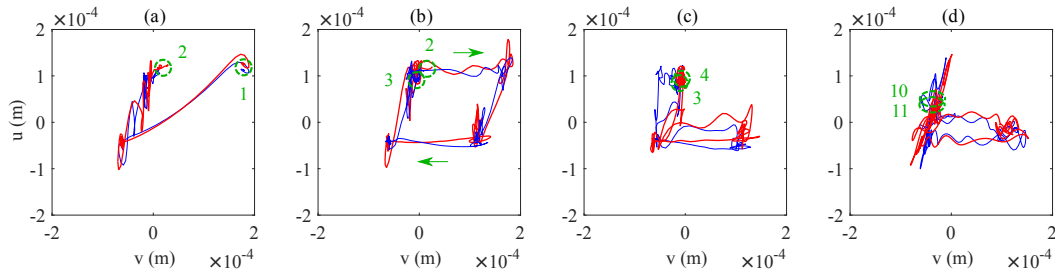


Figure 2: String displacement in the plane ( $Oyz$ ) at  $x=854$  mm and at  $y=845$  mm, showing two  $u$  and  $v$  components. Blue: experiments, Red: numerics for four oscillation periods: 1 to 2 (a); 2 to 3 (b); 3 to 4 (c) and 10 to 11 (d) [1]

In figure 2 is shown an example of a very good agreement between numerical and experimental data measured on an electric bass. The friction force was adjusted empirically. As we can see in this figure, numerical and experimental string displacements overlap at each time step from the first period of oscillation to the tenth after colliding many times with 20 frets.

During the talk, other examples will be shown on the tanpura, the medieval harp and the electric bass to understand their specific sound and to see how numerical simulations can be used as a tool for helping instrument makers to adjust or design musical instruments.

## References

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