Finite elements based reduced order models for nonlinear dynamics of piezoelectric structures

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Abstract This paper presents a general methodology to predict the dynamics of geometrically nonlinear electro-mechanical structures with piezoelectric transducers. Modal Reduced Order Models (ROM) are built using a finite-element software thanks to a non-intrusive strategy. The resulting system is solved with the Harmonic Balance Method coupled to an Asymptotic Numerical Method (ANM). The present study focuses on the computation of the ROM and its validation with experiments on a test structure, exhibiting bent nonlinear modes, internal resonances and nonlinear response under parametric excitation.

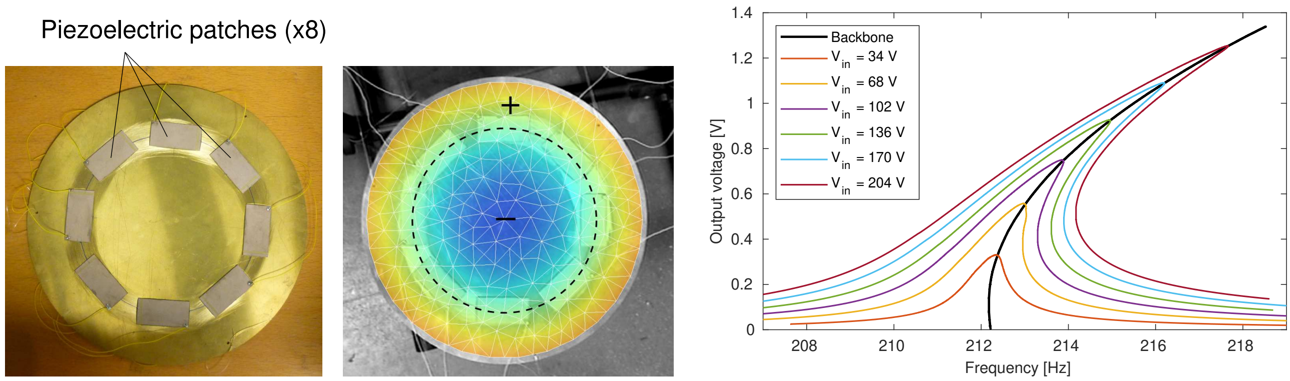

Figure 1: Photograph of the test structure. Deformed shape of the (0,1) mode and experimental nonlinear frequency response in forced and free vibrations (backbone curve) with piezoelectric actuation and detection.

Geometrical nonlinearities, due to large transverse displacements of thin structures, are involved in a large range of applications. Among them, Micro-Electro Mechanical Systems (MEMS) developments has been the focus of numerous studies, whose purpose is to master and use the geometrically nonlinear behaviour (among others, see [5,7,8]). Recent advances in non-intrusive ROM finite element modeling of nonlinear geometric structures offer new perspectives to compute accurate ROM of structures with complex geometries [3]. An application on piezoelectric nanobridges of such a method has been proposed in [2], with a home made finite element code. The purpose of this paper is to extend this approach to a wider range of electromechanical structures, composed of a thin elastic host structures equipped with several piezoelectric patches, for actuation and detection of the vibrations. The modelling proposed here includes: (i) the geometrical nonlinearities (ii) the laminated structure and (iii) the electromechanical transduction with both converse and direct effects.

Following the ideas of [6] for the linear case and [2] for the case with geometrical nonlinearities, we expand the finite element formulation on K eigenmodes of the structures, by writing the displacement vector $\boldsymbol{U}(t) = \sum_{k=1}^{K} \boldsymbol{\Phi}_k q_k(t)$, where $\boldsymbol{\Phi}_k$ is the k-th. eigenvector with the piezoelectric patches in short circuit and $q_k(t)$ the corresponding modal coordinate. It can be shown that it verifies, $\forall k = 1, \dots K$, $\forall p = 1, \dots P$:

$$\begin{cases} \ddot{q}_k + 2\xi_k\omega_k\dot{q}_k + \omega_k^2 q_k + \sum_{i,j=1}^{K} \beta_{ij}^k q_i q_j + \sum_{i,j,l=1}^{K} \gamma_{ijl}^k q_i q_j q_l + \sum_{p=1}^{P} \chi_k^{(p)} V^{(p)} + \sum_{p=1}^{P}\sum_{i=1}^{N} \Theta_{ik}^{(p)} q_i V^{(p)} = F_k, & \text{(1a)} \\ C^{(p)}V^{(p)} - \sum_{k=1}^{K} \chi_k^{(p)} q_k - \sum_{i,j=1}^{K} \frac{1}{2}\Theta_{ij}^{(p)} q_i q_j = Q^{(p)}. & \text{(1b)} \end{cases}$$

In the above equations, P piezoelectric patches have been considered, whose electrical state is defined by $(V^{(p)}, Q^{(p)})$, respectively the voltage between the electrodes and the electric charge contained in one of the electrodes. The above model is composed of four separated parts: (1) the linear part (that depends on the k-th eigenfrequency in short circuit ω_k, the modal damping factors ξ_k and the modal mechanical forcing F_k), (2) the geometrical nonlinear part (with coefficients β_{ij}^k and γ_{ijl}^k), (3) the linear piezoelectric coupling (defined by the coupling coefficients $\chi_k^{(p)}$ between mode k and patch p) and (4) a less classical part stemming from both the geometrical nonlinearities and the piezoelectric coupling (of coefs. $\Theta_{ij}^{(p)}$), introduced in [2] and responsible of parametric excitation effects in thin structures [7].

In this context, we propose an extension of the method introduced in [4] to compute all coefficients of the above ROM and some validations. A first set of validations is obtained by considering theoretical test cases for which analytical models are at hand (such as a hinged-hinged beam with two symmetrically disposed piezoelectric patches that cover its whole length). Then, some experiments are also considered, on a specially designed test structure, composed of a circular brass plate equiped with eight piezoelectric patches (Fig. 1). Using experimental continuation [1], the free (backbone curves / nonlinear mode) and forced vibrations are obtained for the first axisymmetric mode (Fig. 1), for two companion asymmetric modes involved in internal resonance and also for parametric excitation. In all cases, the piezoelectric patches are used for both actuation and detection.

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