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Retrieving highly structured models starting from black-box nonlinear state-space models using polynomial decoupling

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Abstract This work discusses a model reduction technique for polynomial nonlinear state-space models. The reduction proceeds by translating the large coupled polynomials into a low number of univariate polynomial functions.

1 Introduction

The use of nonlinear state-space models as a generic nonlinear model structure has proven useful in a variety of applications over recent years. The downside of flexibility is the size of the models, and the large number of parameters required in their description. Moreover, the generic set of equations rely on large multivariate polynomial functions which are hard to interpret. This work discusses a method that involves the decoupling of the polynomial functions in order to retrieve structured models. The size of the models is reduced and the use of univariate functions provides more insight into the nature of the nonlinearity.

2 Approach

The method consists of three steps:

1. Decouple the multivariate polynomial ($\mathbf{f}$) starting from its first-order derivate sampled in a number of operating points [1],

$$\mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) = \mathbf{W}\mathbf{g}\left(\mathbf{V}^{\mathrm{T}}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{u}(t)\end{bmatrix}\right), \tag{1}$$

   here $\mathbf{g}$ is a univariate vector function of a set of intermediate variables. The nonlinear mappings of $\mathbf{g}$ are referred to as branches.

2. Exploiting linear dependencies amongst branches, their number is reduced in successive steps. Doing so, model complexity is balanced to accuracy.

3. The decoupled function is plugged back into the nonlinear state-space model and nonlinear optimisation is used to ensure good performance.

3 Results on the Silver box system

The Silver box is an electrical implementation of the forced Duffing oscillator. Polynomial nonlinear state-space (PNLSS) modelling [2] results in the following model,

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k) + \begin{bmatrix} e_{11} & \cdots & e_{17} \\ e_{21} & \cdots & e_{27} \end{bmatrix} \begin{bmatrix} x_1^2(k) \\ x_1(k)x_2(k) \\ x_2^2(k) \\ x_1^3(k) \\ x_1^2(k)x_2(k) \\ x_1(k)x_2^2(k) \\ x_2^3(k) \end{bmatrix} \tag{2a}$$

$$y(k) = \mathbf{c}\mathbf{x}(k) + du(k), \tag{2b}$$

where the nonlinear part (in red) is described using 14 parameters. Without loss of performance the model is reduced to the following form,

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k) + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \left[\theta_1 z^3(k) + \theta_2 z^2(k)\right] \tag{3a}$$

$$y(k) = \mathbf{c}\mathbf{x}(k) + du(k), \tag{3b}$$

$$z(k) = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \tag{3c}$$

which uses only 6 parameters (corresponding to only 4 d.o.f) in the nonlinear description. The results on a validation data set are shown below. Results are also presented for the Bouc-Wen system, the Van der Pol system and a Li-Ion battery model.

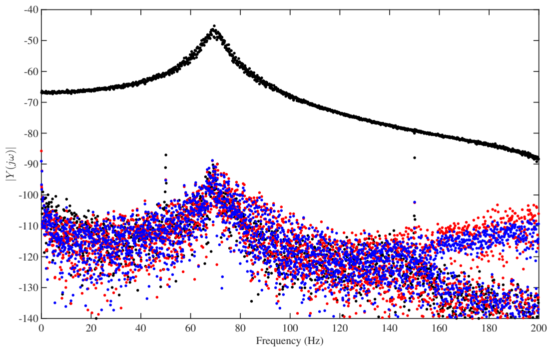

Figure 1: Results of the Silverbox model on a validation data set. Black is the true output, blue is the PNLSS model error, red the reduced model error.

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