Multi-input phase resonance testing of a nonlinear wing-engine structure using control-based continuation

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Abstract Control-based continuation (CBC) is a general and systematic method that proved useful to identify the nonlinear normal modes (NNMs) of conceptually-simple structures directly during experimental tests. The accurate identification of the NNMs of more complex structures can, however, require the application of multiple input forces. CBC is here extended to multi-input experiments and demonstrated on a wing-engine structure.

Nonlinear normal modes (NNMs) are families of periodic responses of the unforced undamped system that have been successfully exploited to interpret the nonlinear dynamic behaviour of forced and damped systems. For instance, NNMs are found to capture the amplitudedependence of the resonance frequency of many harmonically-forced systems, which is valuable from an engineering perspective because this is where displacements are often maximum and the structure is at the greatest risk of failure.

NNMs can be identified experimentally through a phase quadrature condition between the system response and the applied excitation. At quadrature, external input forces exactly counterbalance the internal damping forces and hence the system responds as the underlying conservative system. Experimentally finding and tracking this phase quadrature condition is commonly referred to as phase resonance testing. Identified NNMs can, in turn, be exploited for parameter estimation or compared to theoretical predictions for model updating and validation [1].

The phase resonance testing of complex nonlinear structures raises a number of challenges. The quadrature condition often lies close to a saddle-node bifurcation. As such, reaching quadrature requires to test the physical structure on the verge of instability, i.e. where the system's response is particularly sensitive to perturbations and untimely transition to other behaviours can occur. Another difficulty is that quadrature can give results that are significantly different from the true NNM, in particular, when the excitation applied to the system is limited to one input force, as is frequently used in practice [4].

Control-based continuation (CBC) is a testing method that uses feedback control to change the linearisation of the dynamics such that unstable responses of the underlying uncontrolled system can be made stable. The controller also maintains the experiment around a prescribed operating point, thus avoiding untimely transitions between coexisting behaviours. As such, CBC has the potential to be a general and systematic tool to identify NNMs. The method was, for instance, exploited to track phase quadrature conditions in the forced response of several single-degree-of-freedom systems subjected to external harmonic excitations [2, 3]. Although these successful results attest of the power of this method, CBC has been so far limited to conceptually simple experiments with single-input excitation.

The objective of this work is to further develop CBC such that it can be applied to more complex experiments including multiple inputs. The presence of several inputs offers the flexibility to exploit one or several exciters to control the experiment. The effects of this choice on the controller design and the complexity of CBC are discussed. The presence of detrimental interactions between the different sources of excitation through the tested structure is also analysed.



Figure 1: Nonlinear wing-engine structure used to demonstrate the extension of CBC to experiments with multiple input excitations and perform phase-resonance testing.

To validate the extension of CBC to multiple inputs, CBC is tested on a wing-engine structure (Figure 1). This structure is composed of an aluminum plate to which two masses are suspended through a nonlinear mechanism. A linear state feedback controller designed using optimal control techniques and a local linear model of the experiment is sufficient to apply CBC to this multi-input experiment. This further demonstrates that no sophisticated (nonlinear) control strategies are necessary for CBC to work. CBC is then exploited to carry out multi-input phase resonance tests and extract the NNMs of the structure. NNMs identified using either one or two input forces are compared. Controller invasiveness is analysed in both configurations of the excitation.

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References

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