

# Port-Hamiltonian Representation of Dynamical Systems. Application to Self-Sustained Oscillations in the Vocal Apparatus

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**Abstract** Phonation is a natural example of nonlinear dynamical system with self-sustained oscillations, here resulting from the controlled nonlinear coupling between the deformable vocal folds and the airflow expired from the lungs through the glottis and the vocal tract. As a proof of concept, we propose a minimal model of the full vocal apparatus using the port-Hamiltonian representation that emphasises on the structure of a system (on the separation between the behaviour of the subsystems and their interconnection) and on the power exchanges. Numerical results on bifurcations are qualitatively discussed in relation to voice pathologies.

**Motivations** The physics of voice production has motivated a wide variety of models, from full-featured numerical ones (mainly based on FEM for vocal folds and FVM for the airflow) to reduced order models focusing on the essential phenomenon underlying the phonation. A large body of work in the latter category relies on the description of the glottal aerodynamics from the late 1950 that is based on experiments in rigid static larynx-like ducts. However phonation intrinsically implies the vibration of the vocal folds periodically closing the glottis. There is a significant paradox in those simplified models: the vocal folds move as a consequence of the power exchanged with the glottal flow, but the description of the latter assumes that it does not receive or provide any power to the folds. This power imbalance in the models introduces a bias in the analysis of the coupling occurring in the larynx and of the instability leading to voice production.

**Port-Hamiltonian systems (PHS)** The port-Hamiltonian theory combines the views of the (geometric) Hamiltonian mechanics and of port-based modelling approach, emphasising the separation between the behaviour of components and their interconnection. The *lingua franca* of this theory is energy : port-Hamiltonian systems are open passive systems that can store, dissipate and exchange power with their neighbourhood. Energy-storage is defined by the Hamiltonian  $H(x)$  as a function of state variables  $x$ . This dependency, formulated as the Hamiltonian gradient  $\nabla_x H$ , expresses the *effort* of the energy-storing component. Conversely, the evolution of this component is described by the *flux* variable  $\dot{x}$ . Effort and flux are power dual variables, i.e., they jointly define the *energy flow*  $dH/dt = \dot{x} \cdot \nabla H(x)$ .

Dissipating components are described by dissipation variable  $w$  and their constitutive laws  $z(w)$ , such that the dissipated power is  $\mathcal{P}_{\text{diss}} = w \cdot z(w) \geq 0$ . Finally, external interactions are classically described in terms of inputs  $u$  (also called efforts) and output  $y$  (fluxes) that are dual with respect to the external power:  $\mathcal{P}_{\text{ext}} = y \cdot u$  ( $\geq 0$  when the system yields power).

The geometric structure (Dirac structure, see [1]) accounting for the interconnection of the

subsystems is then described by a matrix  $S$  relating efforts to fluxes:

$$\begin{pmatrix} \dot{x} \\ w \\ y \end{pmatrix} = S(x, w) \begin{pmatrix} \nabla H \\ z(w) \\ u \end{pmatrix}. \quad (1)$$

When the interconnection is conservative, the power balance is ensured provided that the matrix  $S$  is skew-symmetric ( $e \cdot S \cdot e = 0$  for any effort  $e$ ).

**Minimal port-Hamiltonian model of the vocal apparatus** The vocal apparatus is modelled as the interconnection of vocal folds, glottal flow, vocal tract and a subglottal pressure supply. Every component is designed minimally with an emphasis on the dual pairing on the interconnection ports. For instance, each vocal fold is considered to be a single-d.o.f. oscillator with an elastic cover, and is submitted to pressure on the upstream and downstream faces, and interacts with the flow at the glottal face. Reciprocally, the glottal flow is modelled as the simplest kinematics of potential incompressible flow of inviscid air preserving the continuity of the normal velocity on the surface of the vocal folds (see details in Ref. [2]). Downstream the glottis, the flow separates from the folds into a jet that spreads and dissipates its kinetic energy into heat. Finally, the vocal tract is represented as seen by the larynx, i.e. by means of its input impedance describing the acoustic feedback on the larynx.

The full vocal apparatus is obtained by conservative interconnection of the previous subsystems. For the sake of conciseness, the resulting Hamiltonian and matrix  $S$  are not reported here but can be efficiently computed using the Python package PyPHS [3] designed for the symbolic manipulation of PHS systems.

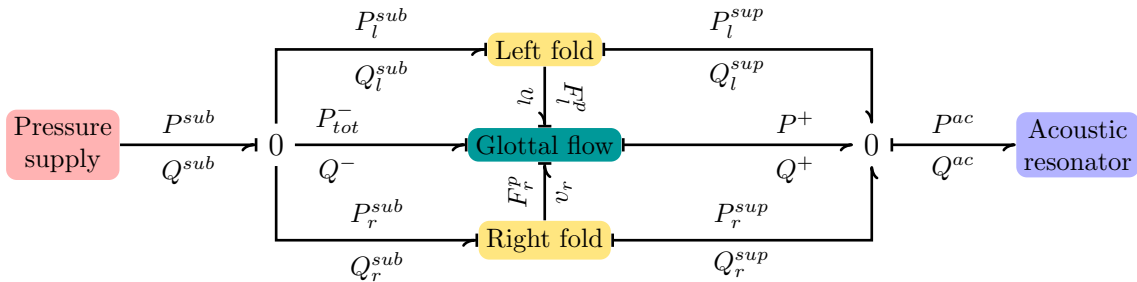


Figure 1: Components of the vocal apparatus. The interconnection takes place via pairs of effort ( $P$ ) and flux ( $Q$ ) variables.

**Numerical results** Time-domain simulations are performed using the numerical scheme designed after the principles of the port-Hamiltonian theory [4], i.e. preserving power balance, but also being more consistent than standard integrators (e.g., RK45, ...). Time-domain simulations and bifurcation analysis evidence the ability of the model to produce oscillating regimes above some pressure threshold, and quasi-periodic vibrations to possible intermittency when vocal folds are detuned.

## References

- [1] A. van der Schaft and D. Jeltsema, *Port-Hamiltonian Systems Theory: an Introductory Overview*, Now Publishers Inc., 2014.
- [2] T. Helie and F. Silva, *Self-oscillations of a Vocal Apparatus: A Port-Hamiltonian Formulation*, 3rd GSI, Paris, 2017.
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- [4] N. Lopes, *Approche passive pour la modélisation, la simulation et l'étude d'un banc de test robotisé pour les instruments de type cuivre*. PhD thesis, UPMC, Paris, 2016.