7th International Conference on Nonlinear Vibrations, Localization and Energy Transfer
July 1-4, 2019 – Marseille, France

Advances to Testing and Model Updating for Geometrically Nonlinear Structures

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Abstract - Model correlation and updating is an important part of the development process of state of the art aircraft, launch vehicles and many other systems. Even though finite element software is highly capable, it still far from reliable when it comes to predicting the actual dynamics of complicated structures such as these. It is necessary to make many approximations around joints, mechanisms, simplifications for details such as sandwich structures, neglecting manufacturing variations, etc.... Furthermore, the current state of the art neglects nonlinearities in the dynamic response. This work seeks to advance the relatively new field of model updating for nonlinear systems. The nonlinear normal modes (NNMs) of the structure are used as a basis for model updating, with new stepped-sine approach used to measure the NNMs and a model updating scheme to update the parameters of a Nonlinear Reduced Order Model (NLROM) for the structure

For the geometrically nonlinear systems that are the focus of this work, the NLROM has the following form [1], where *A* and *B* are the constant coefficients of the quadratic and cubic polynomials respectively, ζ_r and ω_r are the damping ratio and natural frequency of the *r*th mode and $\boldsymbol{\varphi}_r$ is the *r*th mass normalized mode shape.

$$\ddot{\theta}_r + 2\zeta_r\omega_r\dot{\theta}_r + \omega_r^2\theta_r + \theta_r(\mathbf{q}) = \boldsymbol{\varphi}_r^{\mathrm{T}} F(t) \tag{1}$$

$$\theta_r(\mathbf{q}) = \sum_{i=1}^{m}\sum_{j=i}^{m} B_r(i,j)\, q_i q_j + \sum_{i=1}^{m}\sum_{j=i}^{m}\sum_{k=j}^{m} A_r(i,j,k)\, q_i q_j q_k \tag{2}$$

When modeling linear structure such as launch vehicles or aircraft, it is typically necessary to perform tests to update the FEM such that it reflects the correct dynamics, which are captured by ζ_r, ω_r and $\boldsymbol{\varphi}_r$. For a nonlinear structure, the coefficients *A* and *B* that define nonlinearity in the ROM depend on many parameters of the FEM, such as the boundary stiffnesses, material properties, precise curvature of the geometry, etc… and hence we anticipate that it will be necessary to employ testing and model updating to bring the NLROM into agreement with measurements.

The presentation will discuss the authors' latest approach to model updating for these types of structures. The first step involves experimentally estimating the nonlinear normal mode(s) (NNMs) of the system. To do this, the structure is excited near but not precisely at resonance (in order to circumvent difficulty associated with tuning the input so near the point of maximum where the response, where the system is prone to fall off of the resonance. Then the known linear modes of the structure are used with a simplified model to extrapolate to the precise NNM. This approach can speed up stepped-sine testing considerably. The method is illustrated in Figure 1, for real experimental measurements near the first NNM of a nominally flat clamped-clamped beam that is base excited by a shaker. The measurements appear to be very near the NNM because the nonlinear FRFs for this structure are nearly parallel to the actual NNM curve near resonance. The other pane illustrates this for simulated measurements from a curved beam that exhibits both hardening and softening nonlinearities.

Once the NNMs have been measured, the NNMs of the NLROM can be computed and compared. Many iterations may be required to adjust the model parameters until the NNMs come into agreement. This work employs a new algorithm that can significantly accelerate model updating by using a multi-harmonic balance approach in which the gradients of the NNMs with respect to the NLROM parameters are available analytically. Specifically, the gradient of the harmonic amplitudes, **z**, with respect to model parameters, **p**, is given by the following closed form expression in terms of the gradient of the internal forces $\tilde{\mathbf{f}}$ with respect to the model parameters. For an NLROM the parameters are the nonlinear stiffness coefficients, A and B, and so these are known in closed form. The other matrices are part of the harmonic balance method [2] and are similarly known.

$$\left[\frac{\partial \mathbf{z}}{\partial \mathbf{p}}\right] = -\left[\mathbf{A}(\omega) - \boldsymbol{\Gamma}(\omega)^{+}\frac{\partial \tilde{\mathbf{f}}}{\partial \tilde{\mathbf{x}}}\boldsymbol{\Gamma}(\omega)\right]^{-1}\left[\frac{\partial \mathbf{A}(\omega)}{\partial \mathbf{p}}\mathbf{z} - \boldsymbol{\Gamma}(\omega)^{+}\frac{\partial \tilde{\mathbf{f}}}{\partial \mathbf{p}}\right] \tag{3}$$

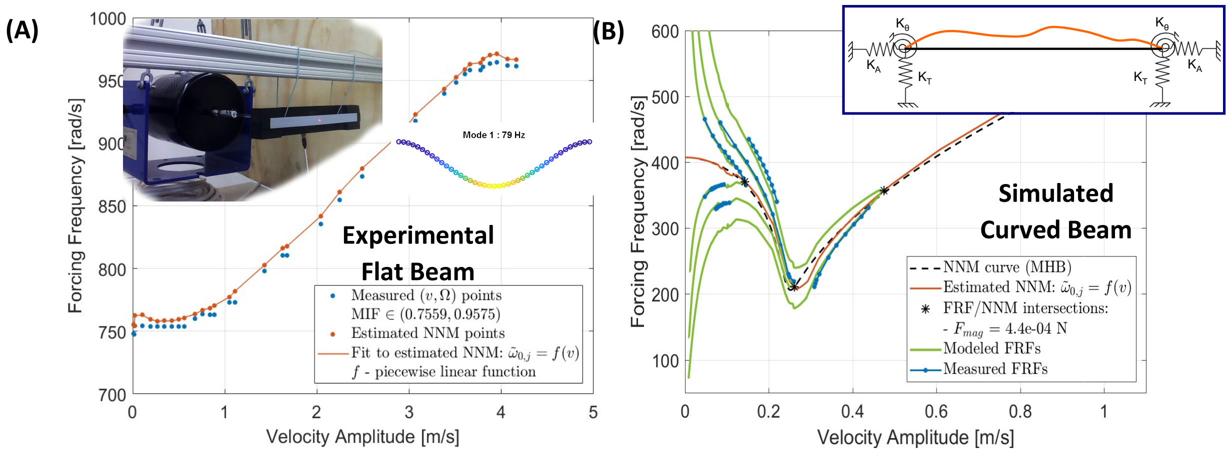

Figure 1. (A) Experimental demonstration of the proposed algorithm to measure NNMs. A series of measurements (blue) are taken near resonance and the single nonlinear resonant mode approximation is used to estimate the NNM (red). (B) Simulated experiments for a curved beam. The nonlinear frequency response is also computed from the measured NNM and compared to simulated measurements (blue).

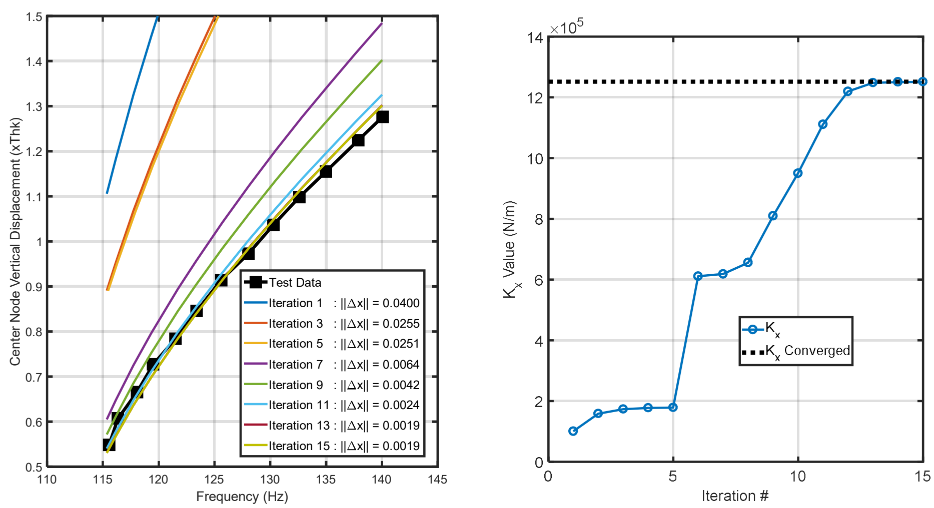

Figure 2. Sample results for nonlinear model updating applied to experimental measurements from a clamped-clamped beam similar to that in Fig. 1. The NNM of the NLROM converges quickly to the measured NNM as the parameters are updated.

References

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