Bistable nonlinear energy sink dynamics via high-dimensional invariant manifolds

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Abstract A bistable nonlinear energy sink, conceived to mitigate the vibrations of a multi-degree-of-freedom host mechanical system, is considered. Under impulsive excitation, the invariant manifolds describing the high amplitude slow dynamics are generalised. Results illustrate that the absorber is generally unable to resonate with more than one mode of the primary system at a time, experiencing instead a sort of "modal cascade" from higher to lower modes.

Nonlinear vibration absorbers, designed to resonate for broad frequency band, have received considerable attention in the last decade. A wide spectrum of nonlinearity sources have been so far addressed, encompassing a variety of excitations, host structure typology, design constraints and objectives [1,2]. Within this context the nonlinear energy sink (NES), consisting of a small mass connected to the primary system by an essential nonlinear spring, has been extensively studied. Recently, the NES capabilities for the mitigation of broadband impulsive energy was studied by the authors exploiting the four-dimensional invariant manifold of a two-DoF host system [3]. Stemming from the latter study, a bistable NES (BNES) connected to a multi-degree-of-freedom (MDOF) system is here considered. Invariant manifolds describing the high amplitude slow dynamics are analytically identified. These consist in high dimensional surfaces, which relate the absorber vibration amplitude to the primary system ones.

The dynamics of the BNES attached to an undamped linear n-DOF primary system is modelled by the following system of differential equations

$$\begin{aligned} &\sum_{j=1}^{n} m_{ij}\ddot{x}_j + \sum_{j=1}^{n} k_{ij}x_j = 0 \quad \text{for } i = 1, ..., n,\ i \neq l \\ &\sum_{j=1}^{n} m_{lj}\ddot{x}_j + \sum_{j=1}^{n} k_{lj}x_j - k_a\left(x_l - x_{n+1}\right) + c_a\left(\dot{x}_l - \dot{x}_{n+1}\right) + k_{nl}\left(x_l - x_{n+1}\right)^3 = 0 \\ &m_a\ddot{x}_{n+1} + c_a\left(\dot{x}_{n+1} - \dot{x}_l\right) - k_a\left(x_{n+1} - x_l\right) + k_{nl}\left(x_{n+1} - x_l\right)^3 = 0, \end{aligned} \tag{1}$$

where $m_{ij} = m_{ji}$ and $k_{ij} = k_{ji}$ are the primary system mass and stiffness matrices terms, m_a is the absorber mass, k_a, c_a and k_{nl} are the absorber negative linear stiffness, linear damping and cubic stiffness coefficients. m_a is assumed small with respect to the primary system masses.

We perform a modal analysis according to the primary system modes and we scale amplitudes by the absorber nonlinearity. Then, aiming at characterizing the behavior of the BNES against impulsive excitation, the invariant manifolds describing the high amplitude slow dynamics are identified, following the procedure adopted in [3]. The obtained invariant manifolds have the form

$$v_{li}^2\omega_{ni}^4 a_i^2 = b_i^2\left(\omega_{ni}^2 + \omega_a^2 + \frac{3}{4}b_i^2 - \frac{3}{2}\sum_{j=1}^{n} b_j^2\right)^2 + 4\zeta_a\omega_a^2\omega_{ni}^2 b_i^2 \quad \text{for } i = 1, ..., n \tag{2}$$

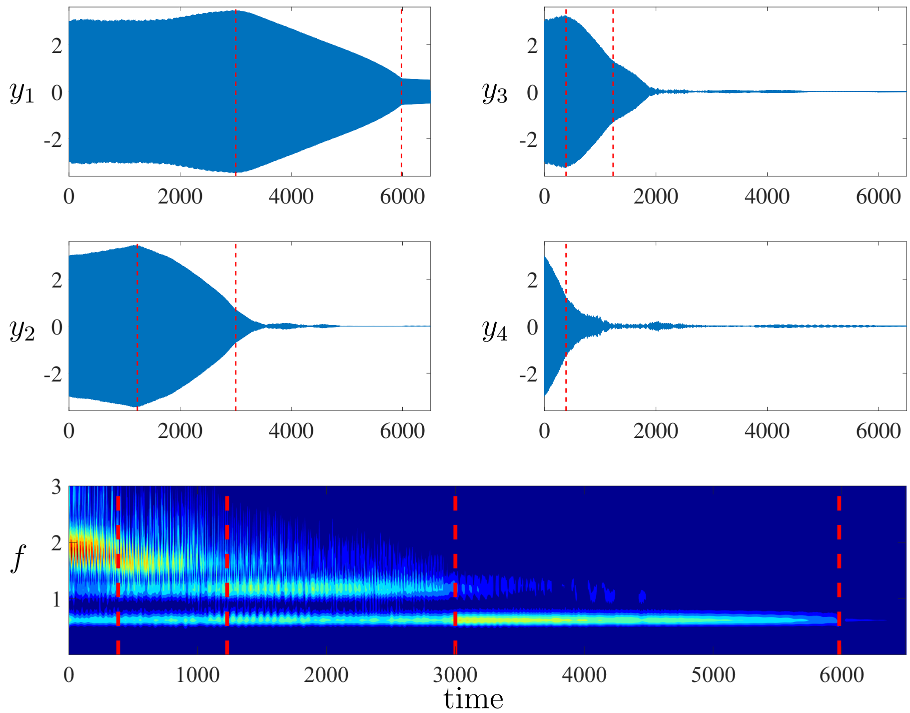

Figure 1: Time series for a 4-DoF primary system with an attached BNES. y_1, y_2, y_3, y_4 indicate modal amplitudes and f the absorber relative displacement wavelet transformation.

where a_i is the vibration amplitude of the i^{th} mode in the primary system, while b_i is the relative vibration amplitude of the absorber with frequency ω_{ni}, ω_{ni} are the primary system natural frequencies, v_{lj} are terms of the transformation matrix utilized for the modal analysis, $\zeta_a = c_a/(2m_a\omega_a)$ and $\omega_a^2 = k_a/m_a$.

Results illustrate that the BNES, although is capable of interacting with all modes of the primary system, it is generally unable to resonate with more than one mode of the primary system at the same time. It experiences instead a sort of "modal cascade", dissipating first energy of higher modes and then of lower ones. Modal interaction between the absorber and more than one mode of the primary system seems to be possible only at very specific energy levels. This is clearly illustrated in Fig. 1 for a 4-DOF primary system. The figure shows how modal energy of the 4^{th} mode is first dissipated (y_4). Then, the BNES disengages from the 4^{th} mode and starts interacting with y_3 ($t \approx 385$, first vertical dashed line), until it disengages also from y_3 and interacts with y_2 ($t \approx 1230$, second vertical dashed line). The process goes on until energy is dissipated on the lowest mode. At $t \approx 6000$ the energy level is so low that the BNES is confined to oscillate in-well.

References

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