7^{th} International Conference on Nonlinear Vibrations, Localization and Energy Transfer
July 1-4, 2019 - Marseille, France

Dynamic instability mitigation by means of nonlinear energy sinks in mechanical systems having one or two unstable modes

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Abstract The presentation is divided into two parts. The first part presents a general method to predict the steady-state regimes of a multi-degree-of-freedom mechanical system (the primary system) having one unstable mode coupled to a set of nonlinear energy sinks (NESs). In the second part the primary system has two unstable modes and it is coupled to one NES. Preliminary numerical results and analytical treatments of this situation are presented.

In the context of passive mitigation of dynamic instabilities, it is now established that the Nonlinear Energy Sinks (NESs) are good candidate to consider especially when low frequency and high level are concerned [1, 2]. The operation of the NESs is based on the concept Targeted Energy Transfer (TET). A basic NES generally consists of a light mass, an essentially nonlinear spring and a viscous linear damper. Because of its essentially nonlinear stiffness, a NES can engage in resonance over a broad frequency range. Whether for a system under impulsive, harmonic or broadband frequency excitation or whether for an auto-oscillating system, TET results from nonlinear mode bifurcations. In general, the phenomenon of TET can be described as a 1:1 resonance capture [3].

This work considers auto-oscillating systems as primary system and it is divided into two parts described below.

1 Prediction of steady-state regimes of a multi-degree-of-freedom unstable dynamical system coupled to a set of nonlinear energy sinks

Here the primary system is a multi-degree-of-freedom (multi-DOF) unstable mechanical systems undergoing cubic nonlinearities and coupled to M NESs. We assume in this section that the primary system has only one mode which can become unstable through Hopf bifurcation.

We propose an analytical method to predict the steady-state regimes of the coupled system. The method begins using the biorthogonal transformation to diagonalize the primary system written in the state-space form. Afterwards, the dynamics of the diagonalized system is reduced keeping only the unstable mode and ignoring the stable modes. Then the slow-flow of the system is obtained using the complexification-averaging (CA-X) method within the assumption of a 1:1 resonance capture around the frequency of the unstable mode. The resulting slow-flow possesses a small parameter related to the mass of the NES and, in the framework of Geometric Singular Perturbation Theory (GSPT) [4], it defines a $(2M, 1)$-fast–slow system. The slow variable characterizes the unstable mode of the primary system whereas the $2M$ fast variables describe the NESs motions (amplitude and phase). It is shown that the Critical Manifold (CM) of the slow-flow can be reduced to a one dimensional parametric curve evolving in a multidimensional space. A similar form of the CM is obtained considering a network of parallel NESs [5]. The knowledge of the stability properties of the critical manifold and the fixed point of the slow-flow (position and stability) makes it possible to predict the response regimes. Finally, the method is applied to the prediction of the steady-state responses of an airfoil undergoing an aeroelastic instability coupled the a set of NESs (from one to four). Theoretical results are compared, for validation purposes, to direct numerical integration of the system. The comparison shows a good agreement.

2 Mitigation by means of nonlinear energy sinks of friction-induced vibrations in a mechanical system having two unstable modes

In this part, the primary system has two unstable modes and, as a first step, it is linear and coupled to only one NES. Our objective is to investigate the solutions in the vicinity of two simultaneous 1:1-1:1 resonances to the natural frequencies of the two unstable modes.

The procedure described above is performed again except that now the two unstable modes must be kept. Moreover, in spite of the presence of the NES coupling, we assume that each variable associated to the primary system (resulting to the biorthogonal transformation) oscillates at one single frequency contrary to the degrees of freedom associated to the NES which must be split as a sum of two terms to capture frequency components with respect to the two unstable modes. Within this assumptions, the CA-X method is applied leading to a slow-flow (in the real domain) which takes the form of a $(4, 2)$-fast–slow system. The slow variables characterize the two unstable modes of the primary system whereas the 4 fast variables describe the NESs motion (amplitude and phase of the two frequency components).

Preliminary analytical results, again based on GSPT, are presented. For example the CM, which appears as a 6-dimensional parametric surface, is determined as well as fixed points of the slow-flow and their stability. These results are validated by comparison to numerical simulation of the system.

References

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