Wave propagation on a beam resting on a unilateral soil

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Abstract The wave propagation problem of an Euler-Bernoulli beam resting on a tensionless foundation is addressed. The exact solution of the governing equation is obtained, selecting among the various mathematical solutions only those having a physical meaning. It is investigated how the stiffness of the unilateral soil influence the wave velocity.

The flexural wave propagation in beams is a classical problem [1], that is characterized by the velocity of propagation $c_{low} = \frac{\sqrt{EJ}}{L\sqrt{\rho A}} 2\pi$, L being the wavelenght. When the beam rests on a bilateral elastic soil, of stiffness k, this velocity changes to $c_{up} = \frac{\sqrt{EJ}}{L\sqrt{\rho A}}\sqrt{4\pi^2 + \left(\frac{kL^4}{EJ}\right)\frac{1}{4\pi^2}}$. This problem has been investigated since long time ago, too [2].

Much less investigated is the problem of a beam resting on a unilateral soil, in which the foundation reach in traction (or compression) only [3], while the problem of wave propagation in this case has been studied in [4], a paper that constitutes the background of this work.

The mechanical problem is illustrated in Fig. 1, and is governed by the equation of motion

$$EJw'''' + \hat{k}w + \rho A\ddot{w} = 0, \tag{1}$$

$$\hat{k} = \begin{cases} k, & w > 0, \\ 0, & w \leq 0, \end{cases} \tag{2}$$

where $w(x,t)$ is the transversal displacement of the Euler-Bernoulli beam, EJ the bending stiffness, ρA the mass per unit length and $\hat{k}$ the stiffness of the foundation. We consider *undamped free* wave propagation.

The solution is sought after in the form

$$w(x,t) = \begin{cases} w_1(x,t) = W_1(\gamma_1 x - \omega_1 t), & \text{in the contact part,} \\ w_2(x,t) = W_2(\gamma_2 x - \omega_2 t), & \text{in the detached part,} \end{cases} \tag{3}$$

where

$$\omega_1 = \sqrt{\frac{EJ\gamma_1^4 + k}{\rho A}}, \qquad \omega_2 = \sqrt{\frac{EJ}{\rho A}}\gamma_2^2, \tag{4}$$

with relevant continuity/periodicity conditions for $x = L_1$ and $x = L_2$.

After long mathematical developments, that are reported in [4] and that require to add a physical admissibility condition to the purely mathematical solution of (1)-(2), it is possible to determine the wave velocity c_{unil} as a function of the soil stiffness k.

The solution is made of many branches, the first three being reported in Fig. 2. The first branch start from 2π for $k = 0$, increases for increasing k, reaches a maximum and then approaches π for k going back to $k = 0$. The other branches, on the other hand, are increasing functions of k, and, apart from the initial part, are always below the case of bilateral soil,

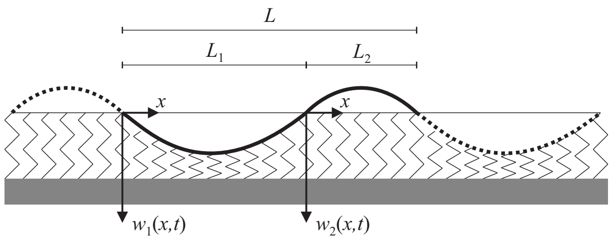

Figure 1: The considered mechanical problem.

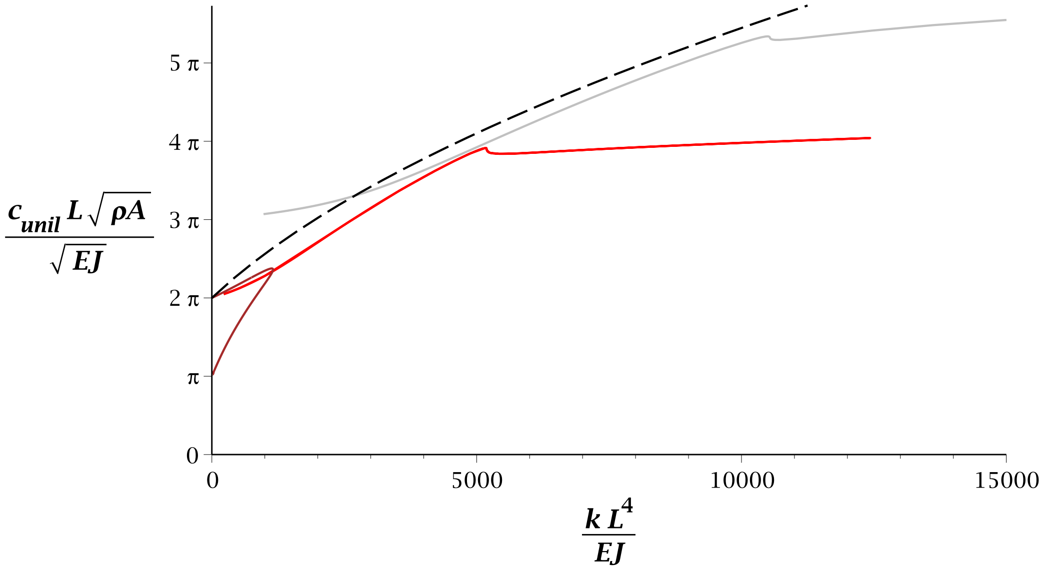

Figure 2: Wave velocity c_{unil} as a function of the soil stiffness. The dash line corresponds to the case of bilateral soil c_{up}.

according to the fact that waves propagate faster on stiffer systems; in fact, the bilateral foundation is stiffer than the unilateral one.

When the solution is close to the dashed line, the wave shape resembles (qualitatively) that reported in Fig. 1, i.e. the detached and in contact parts have about the same length, while far from it one of the two parts becomes predominant.

References

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