Weakly and Strongly Nonlinear Periodic Materials: Tunable Dispersion, Non-Reciprocity, and Device Implications

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Abstract This talk will discuss research aimed at analyzing, simulating, and experimentally exploring weakly and strongly nonlinear acoustic periodic materials. In particular, the talk will focus on the manner in which these materials can be used to create novel wave-control devices for the purposes of wave guiding and filtering.

Weak and strong nonlinearity provide additional design degrees of freedom for achieving novel behavior in periodic elastic media. In weakly nonlinear media, perturbation techniques [1, 2] have recently been employed to uncover amplitude-dependent dispersion and spatial propagation – see Fig. 1 for representative results. These techniques asymptotically expand the displacement field and frequency (or, equivalently, time) and result in a cascading set of linear equations. Removal of secular terms yield updates to the dispersion relationship, while particular solutions at higher orders lead to multiharmonic content. Recent interpretations [3, 4] of these higher-order waves have shown that they can propagate through weakly nonlinear media with little to no generation of higher harmonics, similar to the invariance displayed by solitons.

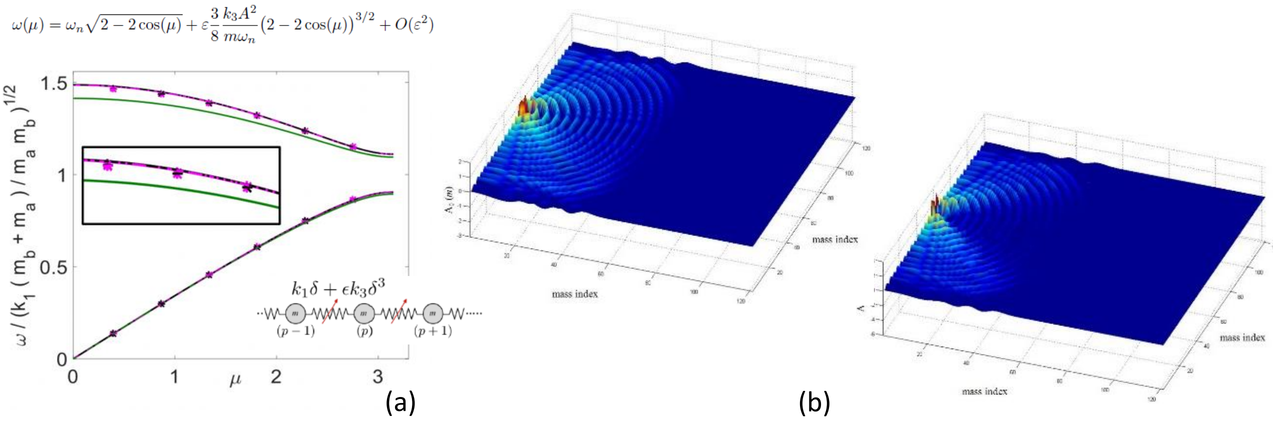

Figure 1: (a) Weakly nonlinear periodic media give rise to amplitude-dependent dispersion and (b) amplitude-dependent spatial dead-zones for (left) small and (right) large amplitude waves.

Experimental confirmation of these and other findings are an open area of investigation. Manktelow *et al.* [5] provided an indirect measurement of amplitude-dependent dispersion in a periodic string by placing evenly-spaced lead masses on a taut string, which was then excited by a shaker and measured using a laser Doppler vibrometer. The taut string exhibits a well-known cubic stiffening, which leads to positive shifts to the string's dispersion with increasing amplitude. The analysis connected the system's natural frequencies to the dispersion relationship in the first Brillouin zone using a phase closure argument. Measurements of the nonlinear backbones then resulted in amplitude-dependent dispersion curves – see Fig. 2. While successful, the experiment was limited to relatively small motions due to the string's tendency to exhibit a whirling instability. Experimental studies have yet to appear which confirm the other richness observed both analytically and numerically.

(a) Mass element (b) Apparatus (c) Results

Figure 2: Experimental apparatus and results: (a) lead bead and steel wire, (b) periodic string fixed to two upright aluminium beams, (c) experimentally measured backbone curve AB (black) and theoretical backbone curve for a simplified model (red).

Strongly nonlinear periodic systems exhibit additional advantageous behavior, particularly as concerns non-reciprocal wave propagation. Boechler *et al.* [6] showed that a granular chain with a defect near one end could passively break reciprocity due to a bifurcation involving the defect mass. More recently, strongly nonlinear systems incorporating hierarchical scales and asymmetry have been shown theoretically and experimentally to passively break reciprocity over a large range of impulse-like excitation [7]. Figure 3 illustrates representative results for such systems in which excitation on the left yields propagation, while excitation on the right leads to localization and no propagation. This is an ongoing area of research which is currently being extended to strongly nonlinear systems which passively break plane wave reciprocity.

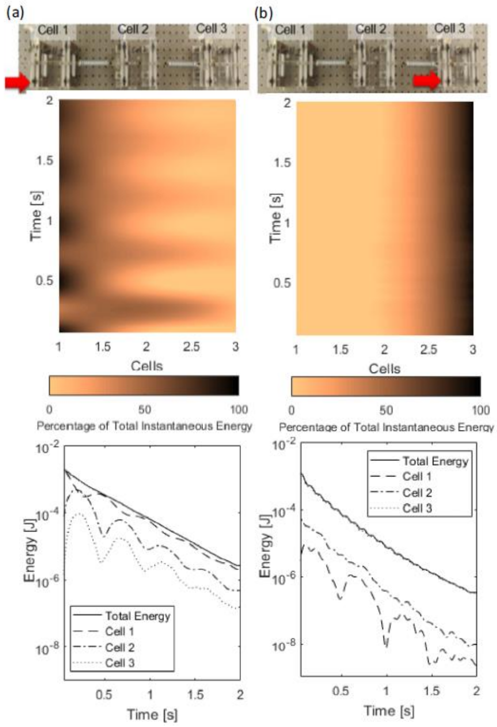

Figure 3: Experimental demonstration of non-reciprocity in a chain consisting of 3 unit cells, each unit cell containing two scales coupled by strongly nonlinear stiffness. Excitation on the left end clearly shows propagation to the right, while excitation on the right remains localized, and no transmission to the left occurs.

References

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