

Stochastic Design Optimization in Nonlinear Vibrations

Keynote Lecture

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Abstract The design optimization of problems involving nonlinear dynamics is tedious because of their inherent sensitivity to uncertainties and the presence of discontinuities. Traditional methods in design optimization are not sufficient to overcome the hurdles encountered in these problems. This lecture will provide an overview of current difficulties in design optimization and uncertainty propagation. It will also present a new framework for the stochastic optimization of nonlinear vibration problems.

Nonlinear dynamics phenomena have been studied and leveraged in several areas of science and engineering. For instance, nonlinear energy sinks (NESs) [1] have become extremely popular in vibration mitigation. Nonlinearities are also important in the development of metamaterials [2] where the tailoring of local nonlinearities can lead to performances and behaviors not encountered in nature. The use of nonlinearities has markedly extended the realm of design possibilities and, in conjunction with strides in additive manufacturing, it is an area that has yet to deliver its full potential.

However, the design, and in particular the optimal design, of dynamical systems exhibiting a nonlinear behavior is particularly tedious for several reasons. First, “jump” behaviors lead to discontinuous responses, which present major hurdles for traditional optimization methods. In the area of nonlinear vibrations, a typical example is provided by the NES efficiency, which is discontinuous when the activation threshold is reached. Related to the presence of discontinuities, the second difficulty is the potentially high sensitivity of the responses to design and loading uncertainties. Finally, high dimensional problems involving computationally intensive function evaluations also represent a major hurdle. Therefore, dedicated design optimization approaches are needed to tackle the specificities of nonlinear vibration problems.

This lecture will discuss the sensitivity of the dynamic behavior to uncertainties by providing examples of jump behaviors that can strongly impact the choice and the efficiency of optimization or uncertainty propagation techniques. For instance, the non-smoothness of the responses prevents the use of gradient-based optimization methods or topology optimization approaches that rely on the computation of adjoint-based sensitivities. Discontinuities also prevent the use of approximations, referred to as surrogates or metamodels (e.g., Kriging, Polynomial Chaos Expansion), that typically enable optimization or uncertainty quantification.

A general stochastic optimization framework for nonlinear dynamic problems, which attempts to address the aforementioned difficulties, will be presented. In particular, a solution scheme to the following optimization problem will be provided [3, 4]:

$$\begin{aligned}
& \max_{\boldsymbol{\mu}^d} \mathbb{E}(F(\mathbf{X}^d, \mathbf{X}^a)) \\
& s.t. \quad \mathbb{P}((\mathbf{X}^d, \mathbf{X}^a) \in \Omega) \leq P_T \\
& \quad \boldsymbol{\mu}_{min}^d \leq \boldsymbol{\mu}^d \leq \boldsymbol{\mu}_{max}^d
\end{aligned} \tag{1}$$

where $\mathbb{E}(F)$ is the expected value of a performance metric F . \mathbf{X}^d are random design variables (e.g., nonlinear stiffness) with hyperparameters $\boldsymbol{\mu}^d$ (e.g., means of distributions) and \mathbf{X}^a are “environmental” random variables (e.g., loading conditions). Ω is a domain of unwanted behaviors (e.g., no NES activation) and P_T is a probability threshold. The key features of the solution scheme are the use of “machine learning” techniques such as clustering and support vector machines (SVM) classifier for the purpose of automatically detecting discontinuities and identifying the boundary, defined through an SVM, segregating various dynamic behaviors. An adaptive sampling scheme, which helps reduce the number of function calls, has been developed for the refinement of the SVM boundary thus enabling the optimization and the calculation of statistical moments and probabilities.

Several examples will be presented: one set of examples will deal with the optimal design of NESs for simple problems as well as for the mitigation of nonlinear aeroelastic vibrations in the sub- and supercritical regimes [5]. Another set of examples will present the design optimization of chains of nonlinear resonators for the mitigation of vibrations using band gap and dissipation effects.

References

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