Nonlinear cable's moving boundary problem: boundary modulation vs. quasi-static drift

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Abstract A nonlinear suspended cable excited by a boundary motion is attacked in two different formulations, i.e., the boundary modulation formulation and the quasi-static drift formulation.

Cable's nonlinear vibration [1] excited by a kinematic boundary motion of a support (deck or tower) is important for cable-stayed structures, and also, theoretically, an interesting fundamental dynamics problem. Indeed, it can be regarded as one of the two key building blocks - the other one being the support dynamics excited by cable tension - for the boundary modulation concept [2], which allows to deal with the two-way (cable-support) coupled problem. This presentation focuses on two different approaches for attacking the first (support-to-cable) coupling problem, i.e., the boundary modulation approach [2] and the quasi-static drift formulation [3][4]. Their conditional equivalence will be analytically established, and differences, limitations will also be reported. Furthermore, an interesting logical connection between the common empirical shape function [3] and the new rationally derived one, will be discussed. A cable with boundary motion is formulated as

$$\ddot{w}+2c\dot{w}-w''-\alpha\left(w''+y''\right)\left[s_d\left(t\right)\cdot\sin\beta_0+\int_0^1\left(y'w'+0.5w'^2\right)\mathrm{d}x\right]=0 \tag{1}$$

where β_0 is boundary motion inclination, $w(x,t)$ and $s_d(t)$ represent the cable's and the support's displacements, respectively. The cable's non-dimensional stiffness is α, and initial sag is $y(x)=4fx(1-x)$, where the sag-to-span ratio is $f=b/l$, with b, l denoting the sag and span. The cable's boundary conditions at x=0 and x=1 are $w\left(0,t\right)=0, w\left(1,t\right)=s_d\left(t\right)\cos\beta_0$. A single-mode cable is chosen, and ω_m is its dominant frequency. The support motion is assumed to be $s_d\left(t\right)=Y_0 e^{\mathrm{i}\Omega_d t}/2+cc.,\ \Omega_d=2\omega_m+\varepsilon^2\sigma_1$, which means the cable is excited parametrically.

<u>In the boundary modulation formulation</u>, one key assumption is introduced, i.e., the moving boundary is too weak to affect cable's linear modal dynamics while its effects are only on cable's higher order dynamics [2], i.e., $O\left(w\right)\sim O\left(\varepsilon\right), O\left(s_d\right)\sim O\left(\varepsilon^2\right)$. Thus, the moving boundary can be transformed analytically to a weak boundary modulation term on cable's slow dynamics through a standard multi-scale expansion ($w\left(x,t\right)=\sum\varepsilon^j w_j\left(x,T_0,T_2\right)$). After finding the solvability condition at the order $O\left(\varepsilon^3\right)$, we get the corresponding reduced model

$$D_2A_m=-\mu_mA_m-\frac{\mathrm{i}}{2\omega_m}\Gamma_mA_m\left|A_m\right|^2-\frac{\mathrm{i}}{2\,\omega_m}\underbrace{\left[\kappa_{S1}\left(\beta_0\right)+\kappa_{S2}\left(\beta_0\right)\right]}_{\kappa_S}\bar{A}_mY_0e^{\mathrm{i}\sigma_1T_2} \tag{2}$$

Here, A_m is cable's dominant modal amplitude appearing in $w_1=A_m\left(T_2\right)\phi_m\left(x\right)e^{\mathrm{i}\omega_mT_0}+cc.$, and the boundary modulation coefficient $\kappa_{S1}\left(\beta_0\right)$, $\kappa_{S2}\left(\beta_0\right)$ can be analytically derived.

In the quasi-static drift formulation, as the cable's standard boundary condition (fixed at both ends) has been relaxed, a modification induced by the moving boundary is introduced, i.e.,

$$w = \psi_0(x) s_d(t) \cos\beta_0 + \phi_m(x) q_m(t) \tag{3}$$

where the quasi-static drift function $\psi_0(x)$ satisfies $\psi_0(0)=0$ and $\psi_0(1)=1$, and the elastic mode shape $\phi_m(x)$ satisfies $\phi_m(0)=\phi_m(1)=0$. Two empirical drift functions are $\psi_0(x)=x$ [3] and $\psi_0(x)=x^2$ [4]. By substituting Eq.(3) into Eq.(1) and using Galerkin discretization technique, we get (similar ordering assumption is also used $O(w)\sim O(q_m)\sim O(\varepsilon), O(s_d)\sim O(\varepsilon^2)$)

$$\ddot{q}_m + 2\mu\dot{q}_m + a_1 q_m - a_2 q_m^2 - a_3 q_m^3 + a_4 \ddot{s}_d + a_5 \dot{s}_d - a_6 s_d + a_7 q_m s_d + O(\varepsilon^4) = 0 \tag{4}$$

Using a proper multi-scale expansion ($q_m(t)=\sum \varepsilon^j q_{mj}(T_0,T_2), j=1,2,3,\cdots$), the reduced model is established through finding the solvability condition

$$D_2 B_m = -\mu B_m - \frac{\mathrm{i}}{2\omega_m}\underbrace{\left(3a_3 + \frac{10a_2^2}{3\omega_m^2}\right)}_{\tilde{\Gamma}_m}|B_m|^2 B_m - \frac{\mathrm{i}}{2\omega_m}\underbrace{\left(-\frac{a_6}{2} - \frac{a_4 a_2 \Omega_d^2 - a_5 a_2}{3\omega_m^2}\right)}_{\tilde{\kappa}_S}\bar{B}_m Y_0 e^{\mathrm{i}\sigma_1 T_2} \tag{5}$$

Here B_m is the cable's dominant modal amplitude appearing in $q_{m1}=B_m(T_2)e^{\mathrm{i}\omega_m T_0}+cc.$ The coefficients $a_1 \sim a_7$ can be analytically derived. A full comparative study will be based upon these two different reduced models (Eq.(2) and Eq.(5)), as illustrated in Fig. 1 below.

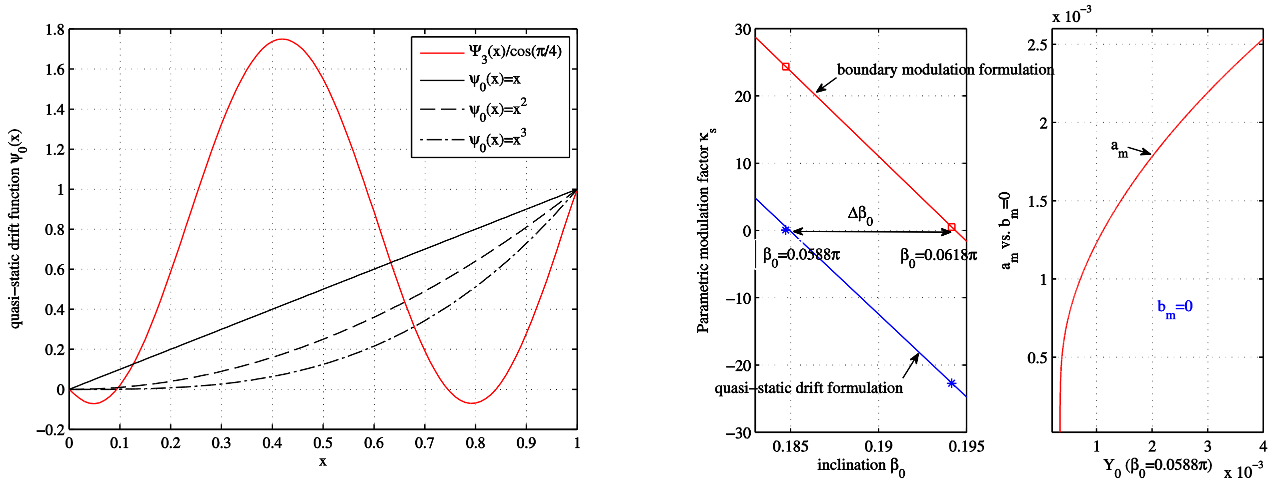

Figure 1: Different shape functions, and two formulation comparisons: μ=0.001, ω_m= 4.21369, σ_1=0.0. $\psi_0=x$ (*), $\psi_0=\Psi_3/\cos(\beta_0)$ (□)

References

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