Capturing nonlinear modal coupling and interactions in reduced-order models

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Abstract A robust methodology for deriving nonlinear reduced-order models from finite-element models would enable powerful nonlinear techniques to be used to analyse large and complex engineering structures. This work highlights why nonlinear modal coupling leads to challenges in establishing such a methodology. Furthermore, using analytical insights gained from considering simple mechanical systems, it is demonstrated how these challenges may be overcome.

Two popular methods for deriving reduced-order models from finite-element models are the Enforced Modal Displacement (EMD) and Applied Modal Force (AMF) methods [1]. These two approaches can lead to significantly different results due to the way in which membrane-type coupling is captured. Specifically, the EMD method often requires that membrane modes are included in the reduced-order model (ROM) [2], whilst the modal parameters calibrated using the AMF method can be sensitive to the magnitude of the force used [3].

In this work, the AMF method is applied to a simple, analytical model of a mass supported by two orthogonal, linear springs, as shown in Figure 1(a). This mass, m, is free to move in two degrees-of-freedom, x and y. The spring parallel to direction x has a length ℓ_1 and a stiffness k_1, whilst the spring parallel to y has a length ℓ_2 and a stiffness k_2. The parameters used here are $m = 1\,\mathrm{kg}$, $\ell_1 = \ell_2 = 5\,\mathrm{cm}$, $k_1 = 100\,\mathrm{N\,m^{-1}}$ and $k_2 = 15000\,\mathrm{N\,m^{-1}}$.

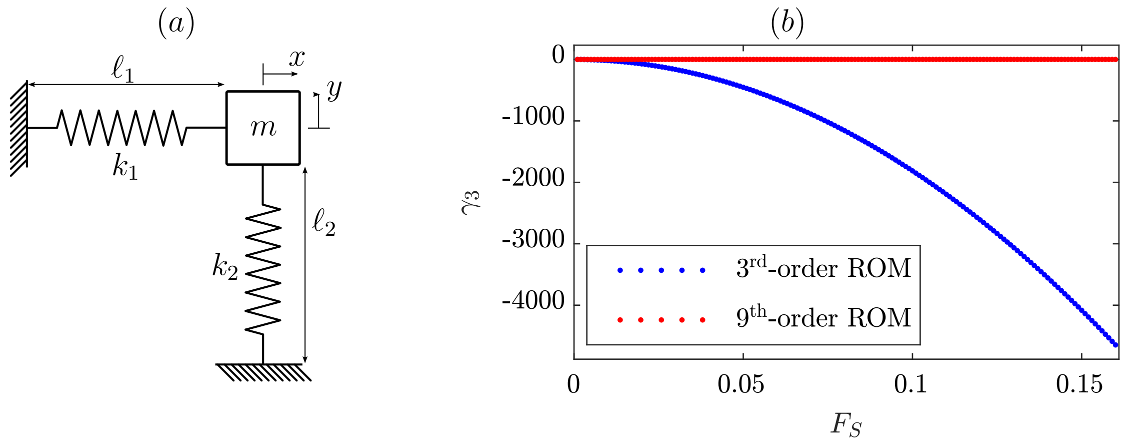

Figure 1: A schematic diagram of the simple oscillator, used to motivate this work, is shown in panel (a). The value of the ROM parameter, γ_3, as the force scale factor, F_S, is varied is shown in panel (b).

The equation of motion of this system may be written

$$\ddot{q}_1 + \omega_{n1}^2 q_1 + 3\alpha_1 q_1^2 + 2\alpha_2 q_1 q_2 + \alpha_3 q_2^2 + 4\beta_1 q_1^3 + 3\beta_2 q_1^2 q_2 + 2\beta_3 q_1 q_2^2 + \beta_4 q_2^3 = f_{q1}\,, \quad (1)$$

$$\ddot{q}_2 + \omega_{n2}^2 q_2 + \alpha_2 q_1^2 + 2\alpha_3 q_1 q_2 + 3\alpha_4 q_2^2 + \beta_2 q_1^3 + 2\beta_3 q_1^2 q_2 + 3\beta_4 q_1 q_2^2 + 4\beta_5 q_2^3 = f_{q2}\,, \quad (2)$$

where the modal coordinates $q_1 = mx$ and $q_2 = my$ have been substituted, and a Taylor expansion, truncated at the third order, has been used to approximate the nonlinear terms. Note that the linear natural frequencies are given by $\omega_{n1}^2 = 100\,\mathrm{rad\,s^{-1}}$ and $\omega_{n2}^2 = 15000\,\mathrm{rad\,s^{-1}}$ – hence the second mode, with a significantly higher frequency, resembles a membrane-type mode. These equations of motion will be treated as the *full-order* system (typically represented by a finite-element method).

To find a reduced-order model describing the dynamics of the first mode of this system, q_1, a suitable parameterised model must be chosen. As the nonlinearity in this full-order system contains quadratic and cubic terms, it appears reasonable to reduce the system to a model containing similar nonlinear terms, i.e.

$$\ddot{q}_1 + \omega_{n1}^2 q_1 + \gamma_2 q_1^2 + \gamma_3 q_1^3 = f_{q1}\,. \tag{3}$$

The AMF method is employed by applying a series of static loads to the first mode of the full-order model (whilst setting the second modal forcing, f_{q2}, to zero) allows the nonlinear parameters of the ROM, γ_2 and γ_3, to be estimated. The blue dots in Figure 1(b) show the estimated value of the cubic parameter, γ_3, as the magnitude of the static loads applied to the full model vary (using scaling F_S). This clearly shows that this nonlinear parameter is sensitive to the magnitude of the applied load, and that the process for calibrating the ROM is not robust.

To understand the cause of this variation, we return to the equation of motion of the second mode, Eq. (2). When the system is static and no load is applied, $\ddot{q}_2 = f_{q2} = 0$, and hence the only remaining variables in Eq. (2) are the modal displacements q_1 and q_2. As such, Eq. (2) represents a constraint between q_1 and q_2, which may be written

$$q_2 = f\left(q_1\right) \approx A_2 q_1^2 + A_3 q_1^3 + \dots\,, \tag{4}$$

where it has been assumed that the function $f(q_1)$ may be approximated using a polynomial. If this polynomial, describing q_2 in terms of q_1, is truncated at the third-order, substituting this into the first equation of motion, Eq. (1), gives

$$\ddot{q}_1 + \omega_{n1}^2 q_1 + \gamma_2 q_1^2 + \gamma_3 q_1^3 + \gamma_4 q_1^4 + \gamma_5 q_1^5 + \gamma_6 q_1^6 + \gamma_7 q_1^7 + \gamma_8 q_1^8 + \gamma_9 q_1^9 = f_{q1}\,. \tag{5}$$

This is representative of a 9^{th}-order ROM (i.e. a ROM where the nonlinearity is expressed up to the 9^{th}-order) which is in contrast to the 3^{th}-order ROM shown in Eq. (4). The variation of the γ_3 parameter of the 9^{th}-order ROM, with the force scale factor F_S, is represented by a red dots in Figure 1(b). This clearly shows that the 9^{th}-order ROM is significantly more robust to variations in the force scale factor than the 3^{th}-order ROM.

When applying the AMF method, the 3^{th}-order ROM is typically adopted. These results demonstrate that, due to the strong coupling that may exist between modes that are well-separated in frequency, the parameters of the 3^{th}-order ROM may vary significantly with the magnitude of the applied load. Furthermore, these results suggest that a higher-order of nonlinearity should be adopted in the ROM in order to account for these coupling effects.

Acknowledgements

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References

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