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A state-space approach to output-only identification of nonlinear systems with load estimation

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Abstract The nonlinear Bayesian state-space model provides a natural framework for modelling of dynamic systems. The particle filter is an efficient tool for working with such systems; however, one of the key challenges is when inputs to a system are non-Gaussian. This paper shows how a latent force approach can be combined with the particle filter in a step towards removing the assumption of Gaussian white noise as an input to the system of interest.

The problem of output-only identification of dynamic systems is by no means a new challenge; there remain many open questions in the realm of linear systems and work on nonlinear systems is, in reality, only just beginning. Methods for identification of linear systems have revolved around the use of modal methods such as stochastic subspace identification [1, 2]. Nonlinear systems fail to exhibit modes in the same way as a linear system (if at all!) contributing to the impossibility of using these methods in the presence of any (except maybe the weakest) nonlinearity. One key assumption made in many of these methods is that the system is under a Gaussian white noise excitation. This assumption becomes stronger and therefore, more problematic with a nonlinear system, where there can exist changes in resonant frequency and other phenomena not seen in linear systems.

Previously, the authors have shown the effectiveness of treating the identification problem as a Bayesian state-space model where the forcing can be treated as a latent state [3]. Using this method, distributions over the unknown parameters of the system are recovered alongside a distribution over possible time histories of the forcing. Here, the extension of this work to a nonlinear example is shown.

A Duffing oscillator is considered, by now a very familiar system to the dynamics community [4] and is used to demonstrate the methods in this work. Moving to this nonlinear system leads to a nonlinear Bayesian state-space model. Unlike the linear case which may be solved with the Kalman filter [5] and Rauch-Tung-Striebel [6] smoothing equations the system is intractable — no closed form solution exists. Instead the system must be approximated, Sequential Monte Carlo methods or particle filters provide an elegant solution for this [7–9],

$$x_t \sim f_\theta\left(x_t \,|\, x_{t-1}, u_{t-1}\right) \tag{1a}$$
$$y_t \sim g_\theta\left(y_t \,|\, x_t, u_t\right) \tag{1b}$$

Here, x_t is a vector of some hidden (latent) states at time t, the evolution of which is governed by $f_\theta\left(x_t \,|\, x_{t-1}, u_{t-1}\right)$. u_t is a vector of 'control' inputs to the model at time t; in structural dynamics this would generally be the force input to the oscillator. These states are related to a vector of observed variables y_t through the probabilistic model defined by $g_\theta\left(y_t \,|\, x_t, u_t\right)$. In this formulation $f_\theta\left(x_t \,|\, x_{t-1}, u_{t-1}\right)$ is the transition density of the model and $g_\theta\left(y_t \,|\, x_t, u_t\right)$ the observation density of the model. The Duffing oscillator defined as,

$$m\ddot{y} + c\dot{y} + ky + k_3 y^3 = F \tag{2}$$

parameterised by its mass m, damping c, linear stiffness k, and cubic stiffness k_3; the oscillator has a displacement, velocity and acceleration $\ddot{y}, \dot{y}$, and y. It is also subjected to an external force F. A Gaussian Process [10] is used as a Bayesian prior over the forcing function in time, this is converted to a state-space representation [11] along with the dynamic system to formulate $f_\theta\left(x_t \,|\, x_{t-1}, u_{t-1}\right)$. Through this reformulation, a joint state-space model between the dynamics and the loading is formed as in [12] but with nonlinear dynamics. The use of particle MCMC [13] is explored to solve this partially-observed nonlinear state-space system and recover the forcing alongside the parameters of the model.

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