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Solitary waves in a non-integrable chain with double-well potentials

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Abstract We study solitary waves in a 1-D lattice of identical masses that are interact by nonlinear springs of double-well potentials. Based on analytical treatment, combined with numerical simulations, we are able to reveal important insights. For example, the solitary wave is indifferent to the energy barrier that separates the two energy wells, and the shape of the wave can be described by means of two scalar properties of the spring potential.

In general, a 1-D lattice with nonlinear interaction between neighbour masses can support propagation of solitary waves. We study special lattice where the neighboring masses interact through springs with double-well potentials, also termed bistable springs. The double-well potential consists of two disjoint intervals of convex energy, “Phase-I” and “Phase-II”, separated by a concave “spinodal” region. This results in a nonlinear, non-monotonous force-strain curve, consisting of two branches with positive stiffness, separated by a branch with negative stiffness. We note the fundamental difference between the system studied here, where double-well potentials govern the *interactions* between neighbours, and the lattice with *onsite* double-well potential, where each mass lies in a double-well potential. In the latter, kink waves (or transition waves), rather than solitary waves, may propagate for very long distances, even in the presence of dissipative mechanisms.

Lattices with double-well-potential interactions have been studied extensively, but not in the context of solitary waves. The Lattice with double-well is prototypical to the behavior of elastoplastic materials, super elasticity, shape-memory effect, plasticity, fracture, hysteresis in material behavior, and more. The *dynamic* behavior of bistable lattices has also been extensively studied, usually by means of modeling inertial dynamics directly, or by assuming simplified evolution strategies or kinetic relations. In addition, the unique nonlinear behavior associated with the double-well-potential has led to the development of a new class of metamaterials, i.e. materials that exploit local instabilities for enhanced performance. These studies have demonstrated the advantages of such architected materials in the context of energy absorption, mechanical/acoustic filtering and tailored bandgaps, transformation of mechanical signals, origami-based metamaterials and other applications.

In a recent work [1], we showed that, governed by the ratio of Phase II to Phase I stiffness, a 1D lattice with double-well-potential interactions may exhibit two fundamentally different dynamic responses following impact. For a softening media (phase-II is softer than phase-I) most of the energy of the impact translates into transition of the first few springs from Phase I to Phase II. However, for a hardening media (phase-II is stiffer than phase-I) a solitary wave, which propagates faster than the speed of sound associated with phase-I, is formed. Further, it was shown that the *height* of the solitary wave is indifferent to the energy barrier separating between phase-I and phase-II. In this study we extend and generalize the results presented in [1], and show that: (i) The entire solitary-wave solution is indifferent to the energy barrier which separates Phase I and Phase II. (ii) The *shape* of the solitary wave is governed by a non-dimensional parameter which reflects the relative significance of the spinodal region in the double-well potential.

The above results are obtained analytically based on padé approximations and are validated by means of extensive numerical simulations that relax the simplifications adopted in the analytical treatment. In particular, the analytical analysis adopts a trilinear approximation for the force-strain behavior of the interaction forces and derives a quasi-continuum approximation based on Padé approximants of the differential operator. These steps enable a straight-forward analytical treatment and the derivation of an explicit solution. In addition, the numerical simulations suggest that the analytical insights apply to bistable lattices with non-trilinear interactions as well. We note that a similar analytical approach has been applied in [2–4] for studying solitary waves in a lattice with bilinear force-strain interactions (convex potentials).

References

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