Nonlinear modal analysis based on complete statistical independence

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Abstract A recent study proposes an extension to the Nonlinear modal analysis framework presented by Worden and Green in [1]. The focus of the current investigation is to advance the method by developing a quantitative measure of modal separation and by considering alternate correlation metrics that are able to detect correlations at any order.

Extensions of linear modal analysis to the nonlinear case have been proposed by several researchers; notably the frameworks from Rosenberg [2] and the geometrically more general work from Shaw and Pierre [3]. A recent paper [1] has put forward a new data-driven approach, leveraging statistical independence to optimise the parameters of a Shaw-Pierre mapping to the nonlinear modal coordinates. For a 2Dof system subject to a cubic transformation, this approach gives a transformation of the form of equation 1 and equation 2 as an objective function.

$$\mathbf{u} = \begin{bmatrix} a_{11} & a_{21} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{21} & b_{31} & b_{41} \\ b_{21} & b_{22} & b_{32} & b_{42} \end{bmatrix} \begin{bmatrix} x_1^3 \\ x_1^2 x_2 \\ x_1 x_2^2 \\ x_2^3 \end{bmatrix} \tag{1}$$

$$j = Cor(u_1, u_2) + \mid \{a_1\} \cdot \{a_2\} \mid + \sum \mid \text{pairwise dot products of the } \{b_i\} \mid \tag{2}$$

In the study, problems arose where the best (lowest cost) transformations found by the heuristic optimisation were not as attractive as less optimal ones when judged by eye. It was suggested that this may owe to the fact that only statistical correlations up to the 2nd order were considered in the optimisation objective function. In order to address this problem, recent work has investigated ways that the results from the forward mapping may be improved.

In [1], the correlation metric used was the Pearson correlation coefficient (Cor). However this formulation is of limited use due to the fact that only correlations up to the 2nd order can be evaluated. The alternate metrics considered by this study are Spearman's rank monotonicity test (Spr) equation, and the Mutual information (MI). These have the advantage of being able to detect monotonic relationships and correlations to any order respectively.

$$\text{Spr}(X, Y) = \frac{\text{Cov}(r_X, r_Y)}{\sigma_{r_X} \sigma_{r_Y}} \tag{3}$$

$$\text{MI}(X, Y) = -\sum_X \sum_Y P(X, Y) \log(P(X, Y)) \tag{4}$$

In order to provide a qualitative measure of mode separation a new approach is presented. First the spectra of the modal responses are estimated by the Welch method. These spectra are then convolved sequentially into a single signal. Finally a thresholding algorithm is used to detect prominent peaks in the convolved spectra. This approach has the desirable property of generating a unimodal signal only when the modal responses are themselves unimodal.

Some initial results are illustrated in figures 1 and 2. By requiring the convolved spectra to have a single peak, better separated modes are identified despite less optimal objective scores.

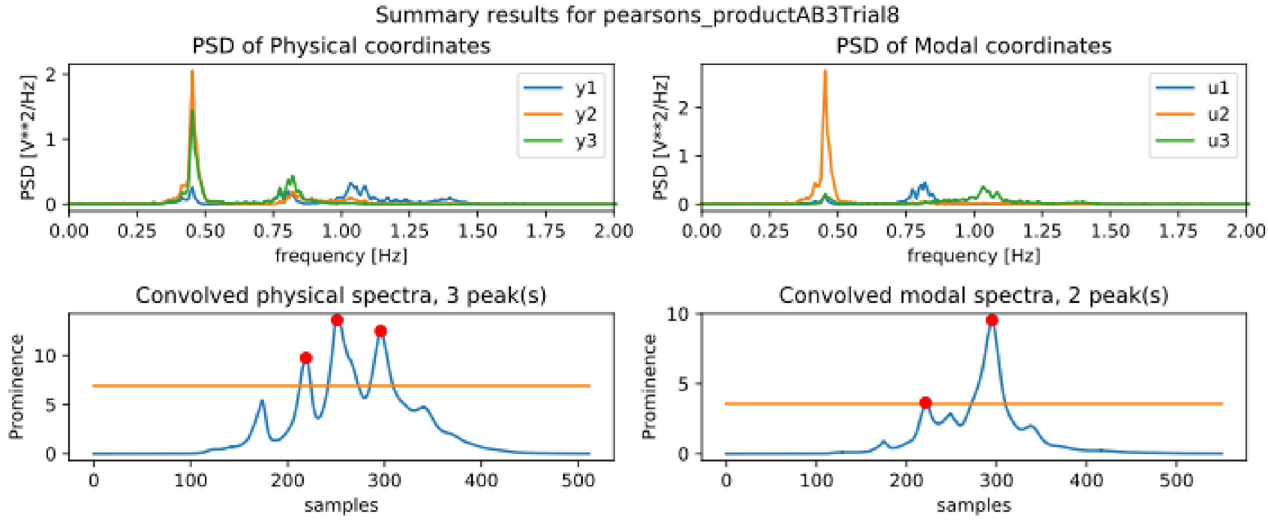

Figure 1: Modal decomposition of 3Dof Duffing system using Pearson's correlation metric - Cubic transformation

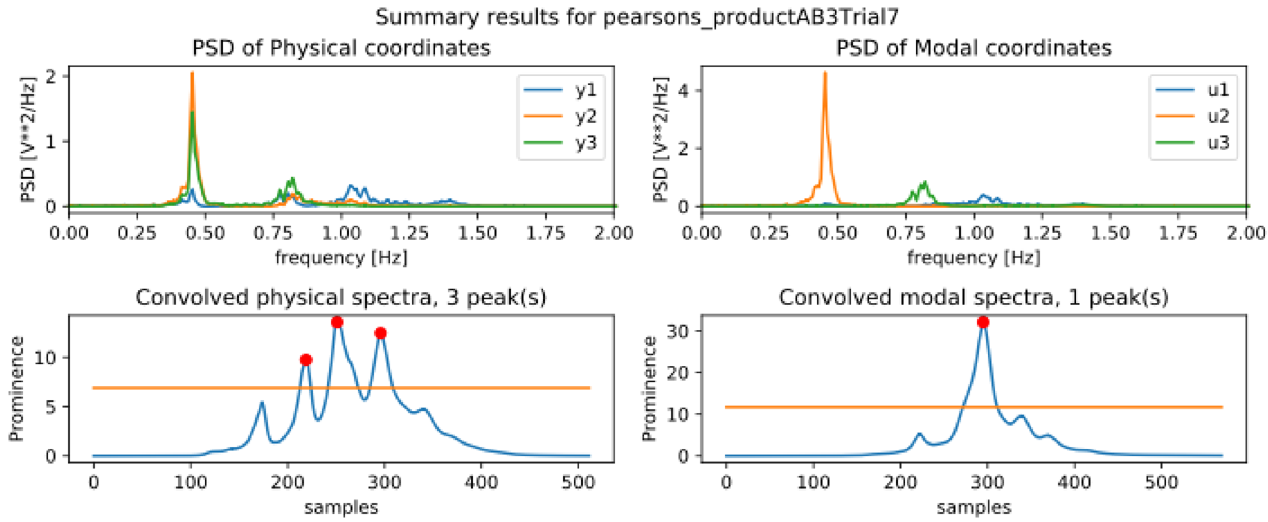

Figure 2: Modal decomposition of 3Dof Duffing system using Pearson's correlation metric - Cubic transformation subject to single peak constraint

Much remains to be done in regards to analysis of the current method. Potential avenues for investigation include; alternate formulations of the objective as a constrained optimisation problem, investigation into ways that computational cost of the metrics might be reduced and wider questions regarding the analytical uniqueness of the NNMs that are computed using this method.

References

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