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Continuation of a physical model of brass instrument: application to trumpet categorization

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Abstract The system formed by the couple {player - trumpet} falls into the class of non-linear dynamical systems likely to be studied using different numerical tools such as numerical continuation methods. In this study we illustrate the interest of this approach for the categorization of Bb trumpets in the space of some performance descriptors obtained from continuation by the ANM method combined to the Harmonic Balance Method (HBM).

The model considered is based on one-dimensional lip model, coupled to the resonator impedance described by a series of complex modes similar to what proposed in [1]. The coupling between the mechanical oscillator and the acoustic resonator is achieved by a Bernoulli flow equation, considering turbulent mixing in the mouthpiece with no pressure recovery [2]. The mechanical and acoustic equations are given in system 1, where y is the vertical lip position (y_0 is the lip position at rest), ω_l, Q_l, μ_l and b the mechanical lip parameters, s_k and C_k with $k \in [1, N]$ the modal parameters of the N resonances of the acoustic impedance of the instrument, Z_c the characteristic impedance, u the volume flow, p the downstream pressure at the input of the instrument (in the mouthpiece), and p_0 the upstream (mouth) static pressure.

$$\begin{cases} \ddot{y}(t) + \frac{\omega_l}{Q_l}\dot{y}(t) + {\omega_l}^2(y(t) - y_0) = \frac{1}{\mu_l}(p_0 - p(t)) \\ \dot{p}_k(t) = Z_c C_k u(t) + s_k p_k(t), \forall k \in [1, N] \end{cases} \tag{1}$$

with $p(t) = 2\sum_{k=1}^{N} \Re(p_k(t))$ and $u = \sqrt{\frac{2|p_0-p|}{\rho}} b \cdot sign(p_0 - p) \cdot \theta(y)$, where $\theta(y) = \frac{|y|+y}{2}$, b is the lip width and ρ is the air density.

The case of a negative opening of the lips is managed by introducing the Heaviside function $\theta(y)$. The modal parameters of the N modes of the impedance are extracted from measured impedances, using the high resolution method ESPRIT [3].

The specificity of the approach proposed in this paper is based on combining the ANM method [4, 5] with the Harmonic Balance Method (HBM) for the search of periodic solutions of the system. The HBM allows to approximate the unknowns by truncated Fourier series. The new unknowns of the problem are the Fourier coefficients of each element of $\mathbf{U}$. For more details about the continuations of periodic solutions using ANM and HBM, the reader is invited to refer to [6].

The calculation of bifurcation diagrams of system 1 is performed using the Matlab library MANLAB developed at LMA[1]. The natural frequency of the lips $f_l = 2\pi\omega_l$ is set to excite the fourth regime of the instrument in open fingering (no valve pressed). In a first step the stationary solution of the system is calculated in order to identify a Hopf bifurcation from which a periodic solution emerges. This periodic solution is then followed by continuation using the method described previously. A bifurcation diagram obtained for a Bb4 on a Bb trumpet is represented in Fig. 1.

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[1] `http://manlab.lma.cnrs-mrs.fr/`

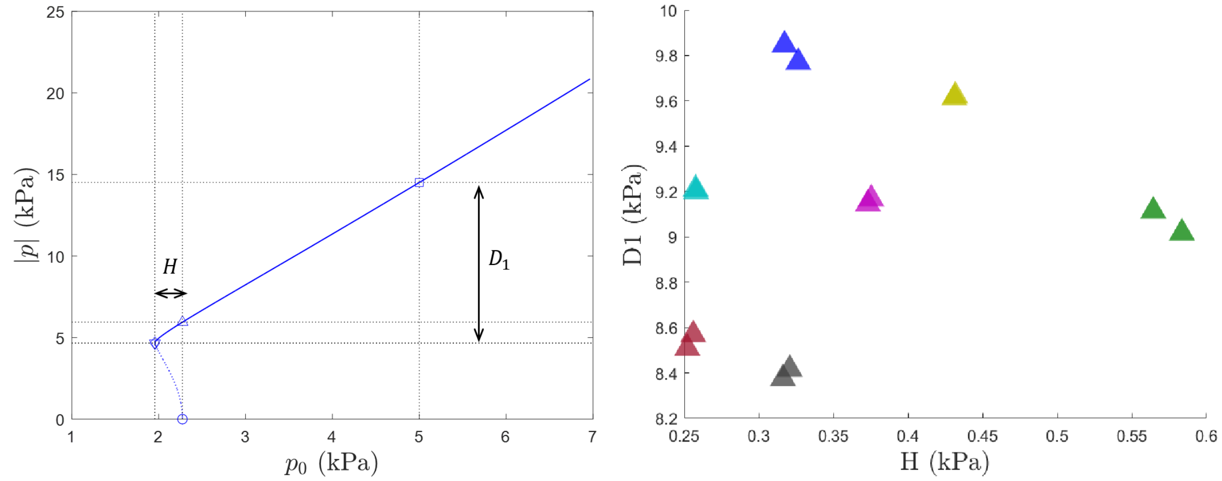

Figure 1: Left: bifurcation diagram of a Bb4 for a Bb trumpet: amplitude of the mouthpiece pressure p as a function of the static mouth pressure p_0. The dotted line indicates unstable portions of the branch, while the solid line indicates the stable branch. Right: categorization of trumpets in the (H, D_1) space. The different colors correspond to the different trumpets. To each trumpet, two points are associated, corresponding to two impedance measurements of the instrument.

The obtained diagram is characterized by an inverse bifurcation that induces an hysteresis H. A quantity D_1 associated to the dynamic range of p can also be defined from the stable part of the solution. By extracting H and D_1 on different trumpets, and locating the corresponding points in the (H, D_1) 2D space, the categorization represented in Fig. 1 is obtained. For each instrument, two impedance measurements are used for calculation. The categorization obtained clearly allows to differentiate instruments in the 2D space, showing the ability of the method to provide discriminating descriptors.

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