Computing nonlinear modes of geometrically nonlinear structures

B. Cochelin, L. Guillot and C. Vergez

Aix Marseille Univ, CNRS, Centrale Marseille, LMA, UMR 7031
Marseille, France
bruno.cochelin@centrale-marseille.fr

Abstract The harmonic balance method is combined to the asymptotic numerical method to compute the nonlinear modes of geometrically nonlinear structural mechanical systems discretized by the finite element method. The method is first described on a toy duffing model and then applied to a beam formulation with large displacements, large rotations but small strain. Various examples computed with the V4 Manlab software are presented.

Let us described the numerical method on a toy duffing equation $\ddot{u} + \lambda\dot{u} + u + u^3 = 0$. Introducing the velocity $v = \dot{u}$ and the auxiliary variable $w = u^2$ the system is recasted as a first order ODE with quadratic nonlinearities

$$\begin{aligned} \dot{u} &= \quad v \\ \dot{v} &= -\lambda v \quad -u * w \\ 0 &= \quad w \quad -u * u \end{aligned}$$

The harmonic balance method aims at determining periodic solutions of the system using Fourier expansion of the unknowns $u(t), v(t)$ and $w(t)$. Denoting $\hat{u}$ the vector of Fourier coefficients of $u(t)$ and ω the angular freuency, the balance of the harmonics yields the following quadratic algebraic system

$$\begin{array}{lcll} \omega\ D\ \hat{u} & = & \hat{v} & \\ \omega\ D\ \hat{v} & = & -\lambda\hat{v} & -\mathrm{conv}(\hat{u},\hat{w})_{same} \\ 0 & = & \hat{w} & -\mathrm{conv}(\hat{u},\hat{u})_{same} \end{array}$$

where D and 'conv' are operators for derivative and convolution (same : means that the output has the same harmonic truncature as the two inputs). A phase condition is added to close the system. The computation of the solution branches of this algebraic system is performed by the so-called asymptotic-numerical method,ie , a high order Taylor series based computation method.

The key point of the method is the quadratic recast of the governing equation. This can be achieved for almost any model by following the procedure presented in [1] . It is illustrated here for a beam model with large displacements and rotations. Let $u(x)$, $w(x)$, $\theta(x)$ denote the displacements and the rotation of the cross-section. By introducing the following auxiliary variables (quadratic equation)

$$\begin{array}{llll} C(x) & = \cos(\theta) \rightarrow dC = -Sd\theta & N(x) & = ESe \\ S(x) & = \sin(\theta) \rightarrow dS = Cd\theta & M(x) & = EIk \\ e(x) & = (1+u') * C + w' * S - 1 & T(x) & = GS\gamma \\ k(x) & = \theta' & F_x(x) & = N * C - T * S \\ \gamma(x) & = -(1+u') * S + w' * C & F_y(x) & = N * S - T * C \\ & & T_2 & = N * \gamma - (1+e) * T \end{array}$$

the governing equations of the beam reads (linear equation)

$$\int_0^L \rho S\ddot{u}\delta u + \rho S\ddot{w}\delta w + \rho I\ddot{\theta}\delta\theta \mathrm{dv} + \int_0^L F_x\delta u' + F_y\delta w' + M\delta\theta' + T_2\delta\theta \mathrm{dv} = 0$$

A finite element discretization is performed with the element descrtibed in [2]. Several nonlinear mode examples will be presented at the conference as in the figure below. The well-known difficulty of dealing with the numerous bifurcations associated with modal interactions will be adressed for NNMs and for forced responses.

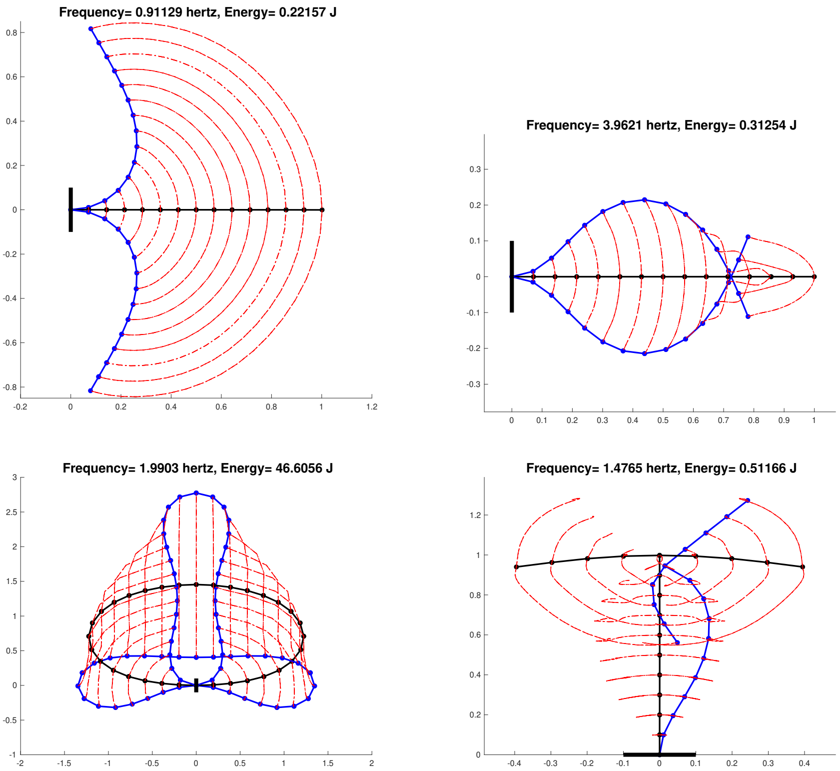

Figure 1: Nonlinear mode periodic motion of various beam structures. In blue : extreme position at $t = 0$ and $t = \frac{T}{2}$, in black : position at $t = \frac{T}{4}$ and $t = \frac{3T}{4}$, in red : trajectory of the nodes. left up : first mode of a clamped-free beam. right up : Second mode of a clamped-free beam with a mass at the end. Left bottom : Second mode of a circular beam. Right bottom : second mode of a T-shape frame. A pure harmonic balance with H=14 harmonics has been used for these computation

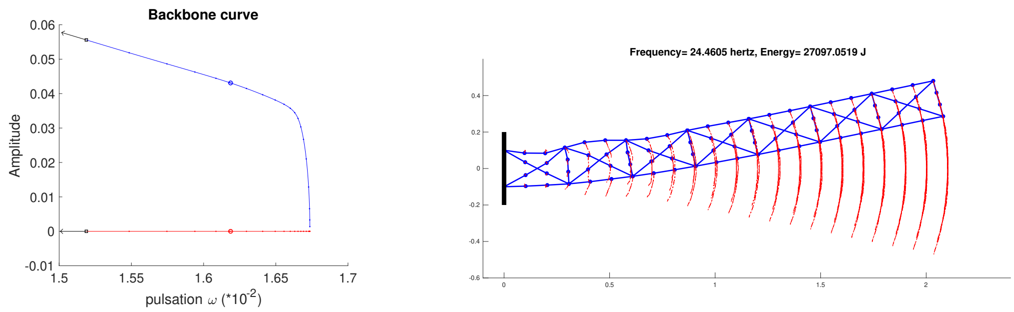

Figure 2: The first nonlinear mode of a this frame structure is softening because of local buckling near the attached end, The model has 258 d.o.f., H=8 harmonics

References

[1] L. Guillot, B. Cochelin and C. Vergez *A generic and efficient Taylor series–based continuation method using a quadratic recast of smooth nonlinear systems*, Inter. J. Numer. Methods Eng, 1-20, 2019.

[2] 0.Thomas, A. Sénéchal, J.F. Deü, *Hardening/softening behaviour and reduced order modeling of nonlinear vibrations of rotating cantilever beams*, Nonlinear Dynamics, 86:1293-1318, 2016.