Nonlinear energy pumping in Acoustics using multistable absorbers

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Abstract This work addresses the development of a vibroacoustics nonlinear absorber based on the concept of a Nonlinear Energy Sink (NES) under multi-stable configuration. Numerical experiments show that adding a bistable property to a NES permits to lower its activation threshold compared to NESs with only one stable position.

Since the seminal papers by Vakakis *et al* [1, 2], energy pumping has become a subject of growing interest. Despite highly efficient energy dissipation, the main drawback is that the higher the frequency of the primary linear system to control, the higher the amplitude for activation of non linear passive dissipation. Recent theoretical and numerical works by Manevitch *et al* [3], Romeo *et al* [4] and experimental work by Mattei *et al* [5] showed that a bi-stable NES (B-NES) provides improved robustness in frequency and amplitude range over existing NESs by lowering the activation threshold. This work aims at improving the absorber developed by R. Bellet *et al* [6]. The developed absorber is made of a high amplitude vibrating membrane that is described by linear and cubic stiffnesses and a dissipation described by linear and quadratic terms. The bistable NES developed in this work can be described by linear quadratic and cubic stiffness terms. The presence of this quadratic damping conplexifies the theoretical analysis but doesn't prevent numerical experiments. The absorber equation developed in [6] is written as

$$m_m\ddot{q}(t) + k_1\left[(1+\chi)q(t) + \eta\dot{q}(t)\right] + k_3(2\eta q^2(t)\dot{q}(t) + q^3(t)) = S_m/(2h)p(t) \quad (1)$$

where $m_m = \rho_m h S_m/3$ is the dynamic membrane mass, S_m its section, η its viscous damping and $p(t)$ is the forcing term. The parameter $\chi = 3R^2e_0/h^2$ is the ratio between the pre-strain e_0 and the (strain) buckling load of the membrane and the coefficients k_1 and k_3 that respectively stand for the linear and nonlinear stiffnesses are defined as $k_1 \approx \frac{1.015^4\pi^5}{12}\frac{Eh^3}{3(1-\nu^2)R^2}$, $k_3 = 8\pi\frac{Eh^3}{3(1-\nu^2)R^2}$. It is worth noting that $1.015^4\pi^5/12 = 8.24 \approx 8$ and thus $k1 \approx k_3$. By denoting $1+\chi = -\zeta$ with $\zeta > 0$ for a buckled membrane, the Eq. (1) is then written as $m_m\ddot{q}(t) + k_1\left[-\zeta q(t) + q^3(t) + \eta(1+2q^2(t))\dot{q}(t)\right] = S_m/(2h)p(t)$. By the change of variable $q_m(t) = \sqrt{\zeta}(y(t)+1)$, one obtains a Helmholtz-Duffing like nonlinear equation for the buckled membrane, very similar to that obtained in [5]

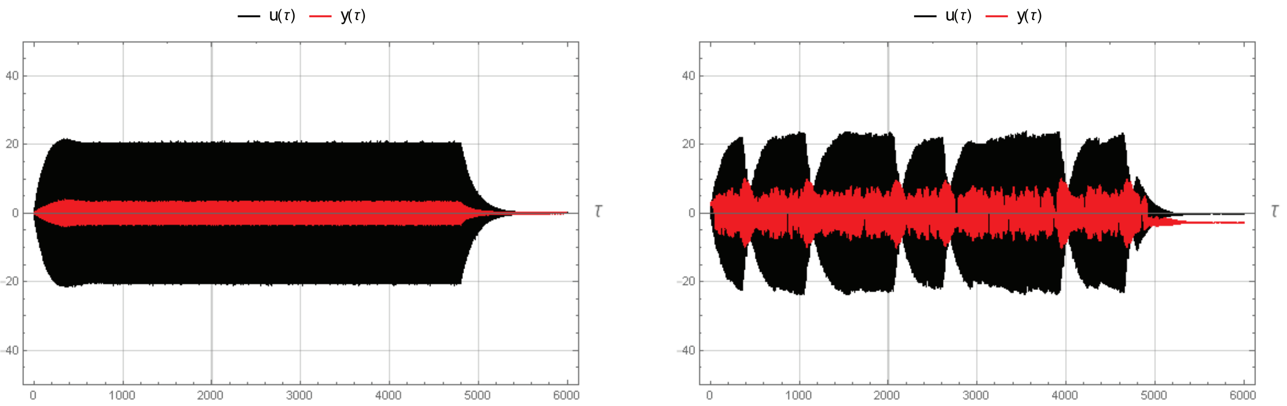
$$\ddot{y}(t)+f_1^2\left[y(t) + 3/2\,y^2(t) + 1/2\,y^3(t) + \eta\left(1/(2\zeta) + (1+y(t))^2\right)\dot{y}(t)\right] = S_m/(2\sqrt{\zeta}hm_m)p(t) \quad (2)$$

where $f_1 = \sqrt{2\zeta k_1/m_m}$ is the linear frequency of the buckled membrane. Such an equation with a non-linearity of the form $y + 3/2y^2 + 1/2y^3$ possesses two stable equilibrium points (0 and -2) and one unstable (-1). We investigated the targeted energy transfer occurring between the acoustic medium (the one-dimensional tube of length L around its first acoustic mode) and the bistable membrane during both the sinusoidal forced regime and the free oscillations as proposed in [6]. Using non-dimensional quantities defined as $\tau = \omega t$, $\omega = c_0\pi/L$, and

$u = u_a 2S_t/(hS_m)$ if u_a is the acoustic velocity inside the tube and S_t its section, with the notations of [6], the two d.o.f. non-dimensional system is given by

$$\begin{aligned}
\frac{d^2u(\tau)}{d\tau^2} + \lambda\frac{du(\tau)}{d\tau} + u(\tau) + \beta\left(u(\tau) - \sqrt{\zeta}(y(t)+1)\right) &= F\cos(\tfrac{\Omega}{\omega}\tau) \\
\gamma\sqrt{\zeta}\frac{d^2y(\tau)}{d\tau^2} + c_1\sqrt{\zeta}\Big[\eta\omega\left(1+2\zeta(1+y(\tau))^2\right)\frac{dy(\tau)}{d\tau} & \\
+\zeta\left(2y(\tau)+3y(\tau)^2+y(\tau)^3\right)\Big] - \beta\left(u(\tau)-\sqrt{\zeta}(y(t)+1)\right) &= 0
\end{aligned} \tag{3}$$

The numerical results for this system are compared to the numerical results for the system (22) given in [6]. The simulation had been obtained for the numerical values: $\lambda \approx 0.014$, $\beta \approx 0.12$, $\omega \approx 545$, $\Omega = 1.06\omega$, $\gamma \approx 1.73$, $\zeta = 10$, $\eta = 10^{-4}$, $c_1 = 0.04$, $c_3 \approx 0.036$, $f_1 = 57$ and $f_1/f_0 \approx 4.4$. The last three coefficients are used for the Bellet's model.The results are presented in Figure 1. It is worth noting that the classical membrane do not show any particular modulation while the bistable membrane show strongly modulated response which is characteristic of dissipation by NES. As the forcing are identical in the two cases, the threshold for energy pumping has been lowered by the bi-stable membrane.



(a) (b)

Figure 1: Time response of the system (22) in [6] figure (a) and 3 figure (b). The forcing of amplitude $F = 13$ is stopped at $t = 4800$.

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